Generalized fifth-order nonlinear evolution equation for the Sawada-Kotera, Lax, and Caudrey-Dodd-Gibbon equations in plasma physics: Painlevé analysis and multi-soliton solutions

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Abstract

This research aims to investigate a generalized fifth-order nonlinear partial differential equation for the Sawada-Kotera (SK), Lax, and Caudrey-Dodd-Gibbon (CDG) equations to study the nonlinear wave phenomena in shallow water, ion-acoustic waves in plasma physics and other nonlinear sciences. The Painlevé analysis is used to determine the integrability of the equation, and the simplified Hirota technique is applied to construct multiple soliton solutions with an investigation of the dispersion relation and phase shift of the equation. We utilize a linear combination approach to construct a system of equations to obtain a general logarithmic transformation for dependent variable. We generate one-soliton, two-soliton, and three-soliton wave solutions using the simplified Hirota method and showcase the dynamics of these solutions graphically through interaction between one, two, and three solitons. We investigate the impact of the system's parameters on the solitons and periodic waves. The SK, Lax, and CDG equations have a wide range of applications in nonlinear dynamics, plasma physics, oceanography, soliton theory, fluid dynamics, and other sciences.

Keywords: Fifth-order equation; Painlevé analysis; Simplified Hirota method; Multiple soliton solutions; Logarithmic transformation.
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1. Introduction

Obtaining the exact solutions for nonlinear partial differential equations (PDEs) [1–9] have captured the attention of numerous researchers, where they have made use of several methodical approaches to achieve the analytic solutions for nonlinear PDEs as multiple solitons, breather, lump solution, kink solitary wave, rogue wave, and others. In recent years, a wide range of techniques [1–19] have been established to understand the different aspects of these analytic solutions such as the Hirota bilinear method, Darboux transformation, simplified Hirota method, Bcklund transformation, Lie symmetry analysis, Pfaffian technique, Inverse scattering method, and several other methods.

Various novel forms of nonlinear PDEs [10–26] have arisen over time, each with distinctive features in soliton theory to learn about their characteristics, and features. Also, new concepts and theories are being implemented in order to produce substantial results. The higher-order nonlinear PDEs are used

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in many fields of science and engineering to explain the dynamical behavior of various physical phenomena such as population dynamics, mechanical vibrations, time-delay processes, astrophysics heartbeat, electronic circuits, nonlinear waves, laser physics, plasma physics, and other nonlinear sciences.

The simplified Hirota method, a simplified version of Hirota bilinear technique [27–29] was established by Hereman et al [30]. This methodology has been widely proposed to describe and apply approaches that find their relevance in dealing with non-linear integrable PDEs. The above said method gives practical outcomes in the form of multiple soliton solutions for a wide range of non-linear PDEs. When compared with the Hirota bilinear technique, simplified Hirota method does not show the dependency on the creation of the bilinear form of a nonlinear PDE, rather it considers soliton solutions as an expression of polynomials of the exponential functions.

Painlevé analysis [31–33] is believe to be an essential tool to understand the integrability of nonlinear PDEs, which exist in several branches of nonlinear sciences. The integrable nonlinear PDEs have significant scientific properties that provide an understanding of the quantitative and qualitative nature of these equations such as Lax pair, conserved quantities, bi-Hamiltonian structure, N-soliton solutions, and others.

In the previous literature review [34–42], we found fifth-order Sawada-Kotera, Lax, and Caudrey-Dodd-Gibbon equations to study the ion-acoustic waves in plasma physics, the nonlinear waves in shallow water, vibrations in mechanical engineering and other nonlinear domains, defined as

$$u_t + (\frac{5}{3}u^3 + 5uu_{2x})_x + u_{5x} = 0, \tag{1}$$

$$u_t + (10u^3 + 10uu_{2x} + 5u_x^2)_x + u_{5x} = 0, \tag{2}$$

$$u_t + (60u^3 + 30uu_{2x})_x + u_{5x} = 0, \tag{3}$$

respectively. All three equations are of the KdV-type and have similar qualities in that they are integrable, and also, have multiple soliton solutions. These equations are nonlinear because (1) and (3) contain two nonlinear terms and (2) contains three nonlinear terms. To get the multi-soliton solutions of these equations by applying simplified Hirota method, we apply logarithmic transformation for dependent variable u as $u = R(ln\Gamma)_{2x}$ with Γ as an auxiliary function and R as a constant. Because all three equations have distinct values for the constant R, the generalization of these equations into a general equation may become tedious. As a result, we use a linear combination approach to generate a general logarithmic transformation that meets the requirements for all three equations (1), (2), and (3).

In this article, we explore a generalized fifth-order equation for the SK, Lax, and CDG equations, which is as follows:

$$u_t + (au^3 + buu_{2x} + cu_x^2)_x + u_{5x} = 0, \tag{4}$$

where a, b, and c are constants. Equation (4) may reduced into the three equations (1), (2) and (3) for $\{a = \frac{5}{3}, b = 5, c = 0\}$, $\{a = 10, b = 10, c = 5\}$, and $\{a = 60, b = 30, c = 0\}$, respectively. The integrability of the equation (4) will be established using required conditions for the constants a, b, and c, and further, it will be demonstrated that for specified values of a, b, and c, multi-soliton solutions can be produced.

Wazwaz [34–37], Hu [38], Zhang [39], and other researchers [40–42] have examined SK, Lax, and CDG equations on an individual aspect but none has formulated the generalized form for these equations. Our research work investigates the generalized form for these equations as well as checks the integrability of this generalized equation using Painlevé analysis. The primary goal of this article is to ensure that the governing equation is integrable using Painlevé analysis, and by applying the simplified Hirota method, we obtain one-soliton, two-soliton, and three-soliton solutions with their interactions.

The manuscript is structured as follows: In section 2, we use Painlevé analysis to check the integrability of the derived equation. Section 3 determines the general logarithmic transformation for the established equation. In Section 4, we use the simplified Hirota method to construct multi-soliton solutions, and Section 5 concludes our investigation and findings.

2. Painlevé Analysis

Painlevé analysis [31–33] is a widely used analytical approach for testing the integrability of the nonlinear PDEs. In this investigation, we seek a Laurent series expansion as a solution of the equation (4), about a singular manifold $\Psi(x,t)$ as

$$u(x,t) = \sum_{s=0}^{\infty} u_s(x,t)\Psi^{s-\gamma}, \tag{5}$$

where γ is a non-negative integer, and $u_s(x,t); s = 0, 1, 2, ...$, are the functions of x and t. By substituting the equation (5) into equation (4), and equating the most dominant terms, we get

$$\gamma = 2,$$

and leading order behaviors with corresponding resonances s are

$$u_0(x,t) = -\frac{3b\Psi_x^2 + 2c\Psi_x^2 \pm \sqrt{(-120a + 9b^2 + 12bc + 4c^2)\Psi_x^4}}{a}; \quad s = -1, 6, \Delta, \tag{6}$$

where Δ is a set of resonances depending on the constants a, b, and c, which is investigated as
(1) For SK equation with $a = \frac{5}{3}, b = 5, c = 0$

$$u_0 = -6\Psi_x^2; \quad s = -1, 2, 3, 6, 10,$$
$$u_0 = -12\Psi_x^2; \quad s = -1, 5, 6, 12.$$

(2) For Lax equation with $a = 10, b = 10, c = 5$

$$u_0 = -2\Psi_x^2; \quad s = -1, 2, 5, 6, 8,$$

$$u_0 = -6\Psi_x^2; \quad s = -1, 6, 8, 10.$$

(3) For CDG equation with $a = 60, b = 30, c = 0$

$$u_0 = -2\Psi_x^2; \quad s = -1, 5, 6, 12,$$

$$u_0 = -\Psi_x^2; \quad s = -1, 2, 3, 6, 10.$$

In all of the above expressions, the resonance -1 conforms to the irrational choice of singular manifold $\Psi(x,t) = 0$ and we observed explicit expressions for $u_j; j = 1, 2, ...$, with some u_j as arbitrary functions. Also, compatibility conditions for the resonances s, are satisfied identically which implies that equation (4) passes the Painlevé test for complete integrability with the restriction on the parameters a, b and c, and therefore depends on the parameters.

3. Logarithmic transformation

We consider the phase variable Φ_i as

$$\Phi_i = \kappa_i x + \omega_i t, \tag{7}$$

where κ_i are the constants and ω_i is the dispersion. Substituting $u(x,t) = e^{\Phi_i}$ in the linear terms of the equation (4) and solving for ω_i, we get the dispersion as

$$\omega_i = -\kappa_i^5. \tag{8}$$

Considering logarithmic transformation for dependent variable as

$$u = R(ln\Gamma)_{2x}, \tag{9}$$

where $\Gamma = \Gamma(x,t)$ and R is a constant. By considering the function $\Gamma = 1 + e^{\Phi_i}$ and substituting it into equation (4), and solving for R, we get as follow

$$R = 6 \quad \texttt{for} \quad a = \frac{5}{3}, b = 5, c = 0,$$

$$R = 2 \quad \texttt{for} \quad a = 10, b = 10, c = 5,$$

$$R = 1 \quad \texttt{for} \quad a = 60, b = 30, c = 0.$$

Now creating the system of equations using linear combinations of $\{a, b, c\}$ with the variables $\{p, q, r\}$ with respect to R as

$$\begin{aligned} \frac{5}{3}p + 5q &= 6, \\ 10p + 10q + 5r &= 2, \\ 60p + 30q &= 1. \end{aligned} \tag{10}$$

On solving the system (10) for p, q and r, we get

$$p = -\frac{7}{10}, q = \frac{43}{30}, r = -\frac{16}{15}.$$

So, we can have general value for $R = pa + qb + rc$ as

$$R = -\frac{7}{10}a + \frac{43}{30}b - \frac{16}{15}c,$$

and the general logarithmic transformation (9) for the equation (4) will be as

$$u(x,t) = \left(-\frac{7}{10}a + \frac{43}{30}b - \frac{16}{15}c\right)(ln\Gamma)_{2x}, \tag{11}$$

or

$$u(x,t) = \left(-\frac{7}{10}a + \frac{43}{30}b - \frac{16}{15}c\right)\left(\frac{\Gamma\Gamma_{xx} - \Gamma_x^2}{\Gamma^2}\right). \tag{12}$$

4. Simplified Hirota Method: Soliton solutions

4.1. Single-soliton solution

Considering the function Γ in Eq. (12) as

$$\Gamma(x,t) = 1 + e^{\Phi_1} = 1 + e^{\kappa_1 x + \omega_1 t}. \tag{13}$$

The following can be deduced easily from the equation (13)

$$\Gamma_x = \kappa_1 e^{\kappa_1 x + \omega_1 t}, \tag{14}$$

$$\Gamma_{xx} = \kappa_1^2 e^{\kappa_1 x + \omega_1 t}. \tag{15}$$

On putting the values from the equations (13), (14) and (15) into equation (12), we get single soliton solution which is remarkably seen as:

$$u(x,t) = -\left(\frac{7}{10}a - \frac{43}{30}b + \frac{16}{15}c\right)\frac{e^{x\kappa_1 + t\kappa_1^5}\kappa_1^2}{(e^{x\kappa_1} + e^{t\kappa_1^5})^2}. \tag{16}$$

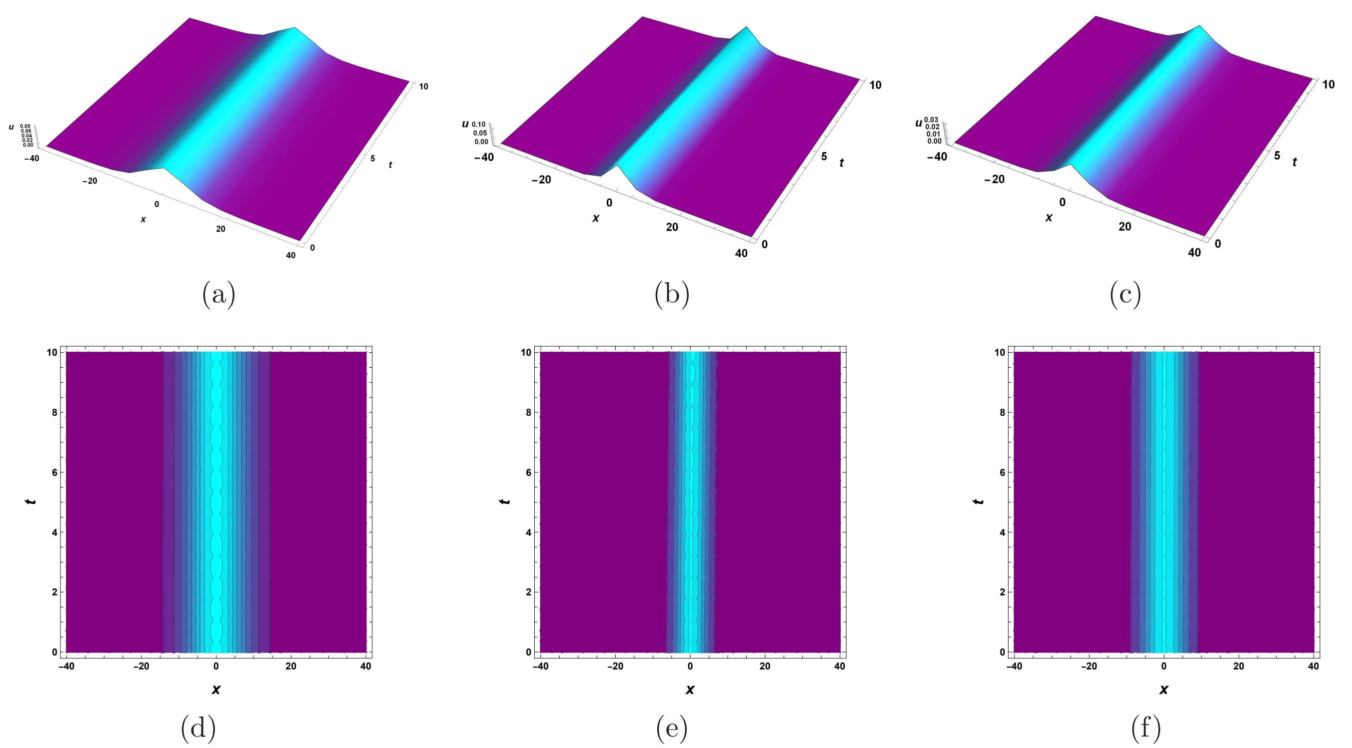

Figure 1: Single-soliton solutions via (12) for the function (13) with values: **(a)** For SK equation: $\kappa_1 = 0.25, a = \frac{5}{3}, b = 5, c = 0$; **(b)** For Lax equation: $\kappa_1 = 0.5, a = 10, b = 10, c = 5$; and **(c)** For CDG equation: $\kappa_1 = 0.35, a = 60, b = 30, c = 0$. **(d-f)** portrays contour plots for **(a-c)** in xt-plane.

4.2. Two-soliton solution

In order to accomplish two-soliton solution, we assume the function Γ as

$$\Gamma(x,t) = 1 + e^{\Phi_1} + e^{\Phi_2} + h_{12}e^{\Phi_1+\Phi_2}, \tag{17}$$

where h_{12} is the dispersion coefficient which can be determined by substituting Γ and its derivatives from equation (17) into the equation (12). Symbolic computation are used to resolve for the values of h_{12}, as showcased below:

$$\begin{aligned}
h_{12} &= \frac{(\kappa_1-\kappa_2)^2(\kappa_1^2-\kappa_1\kappa_2+\kappa_2^2)}{(\kappa_1+\kappa_2)^2(\kappa_1^2+\kappa_1\kappa_2+\kappa_2^2)} \quad \texttt{for} \quad a=\frac{5}{3}, b=5, c=0,\\
h_{12} &= \frac{(\kappa_1-\kappa_2)^2}{(\kappa_1+\kappa_2)^2} \quad \texttt{for} \quad a=10, b=10, c=5,\\
h_{12} &= \frac{(\kappa_1-\kappa_2)^2(\kappa_1^2-\kappa_1\kappa_2+\kappa_2^2)}{(\kappa_1+\kappa_2)^2(\kappa_1^2+\kappa_1\kappa_2+\kappa_2^2)} \quad \texttt{for} \quad a=60, b=30, c=0.
\end{aligned}$$

Now creating the system of equations using linear combinations of $\{a,b,c\}$ with the variables $\{p,q,r\}$ with respect to h_{12} as

$$\begin{aligned}
\frac{5}{3}p+5q &= \frac{(\kappa_1-\kappa_2)^2(\kappa_1^2-\kappa_1\kappa_2+\kappa_2^2)}{(\kappa_1+\kappa_2)^2(\kappa_1^2+\kappa_1\kappa_2+\kappa_2^2)},\\
10p+10q+5r &= \frac{(\kappa_1-\kappa_2)^2}{(\kappa_1+\kappa_2)^2},\\
60p+30q &= \frac{(\kappa_1-\kappa_2)^2(\kappa_1^2-\kappa_1\kappa_2+\kappa_2^2)}{(\kappa_1+\kappa_2)^2(\kappa_1^2+\kappa_1\kappa_2+\kappa_2^2)}.
\end{aligned} \tag{18}$$

On solving the system (18) for p, q, and r, we get

$$\begin{aligned}
p &= -\frac{1}{10}\frac{(\kappa_1-\kappa_2)^2(\kappa_1^2-\kappa_1\kappa_2+\kappa_2^2)}{(\kappa_1+\kappa_2)^2(\kappa_1^2+\kappa_1\kappa_2+\kappa_2^2)},\\
q &= \frac{7}{30}\frac{(\kappa_1-\kappa_2)^2(\kappa_1^2-\kappa_1\kappa_2+\kappa_2^2)}{(\kappa_1+\kappa_2)^2(\kappa_1^2+\kappa_1\kappa_2+\kappa_2^2)},\\
r &= -\frac{1}{15}\frac{(\kappa_1-\kappa_2)^2(\kappa_1^2-7\kappa_1\kappa_2+\kappa_2^2)}{(\kappa_1+\kappa_2)^2(\kappa_1^2+\kappa_1\kappa_2+\kappa_2^2)}.
\end{aligned} \tag{19}$$

So, we can have general value for $h_{12} = pa + qb + rc$ as

$$h_{12} = -\frac{(\kappa_1-\kappa_2)^2\{(3a-7b+2c)\kappa_1^2-(3a-7b+14c)\kappa_1\kappa_2+(3a-7b+2c)\kappa_2^2\}}{30(\kappa_1+\kappa_2)^2(\kappa_1^2+\kappa_1\kappa_2+\kappa_2^2)},$$

which can be extrapolated for the auxiliary function

$$\Gamma(x,t) = 1 + e^{\Phi_i} + e^{\Phi_j} + h_{ij}e^{\Phi_i+\Phi_j}, \tag{20}$$

as

$$h_{ij} = -\frac{(\kappa_i-\kappa_j)^2\{(3a-7b+2c)\kappa_i^2-(3a-7b+14c)\kappa_i\kappa_j+(3a-7b+2c)\kappa_j^2\}}{30(\kappa_i+\kappa_j)^2(\kappa_i^2+\kappa_i\kappa_j+\kappa_j^2)}, \qquad 1 \le i < j \le N. \tag{21}$$

Thus, from equation (17) we get

$$\Gamma_x = \kappa_1 e^{\Phi_1} + \kappa_2 e^{\Phi_1} + h_{12}(\kappa_1+\kappa_2)e^{\Phi_1+\Phi_2}, \tag{22}$$

$$\Gamma_{xx} = \kappa_1^2 e^{\Phi_1} + \kappa_2^2 e^{\Phi_1} + h_{12}(\kappa_1+\kappa_2)^2 e^{\Phi_1+\Phi_2}. \tag{23}$$

The substitution of the equations (17), (22) and (23) into equation (12), results two soliton solution of equation (4).

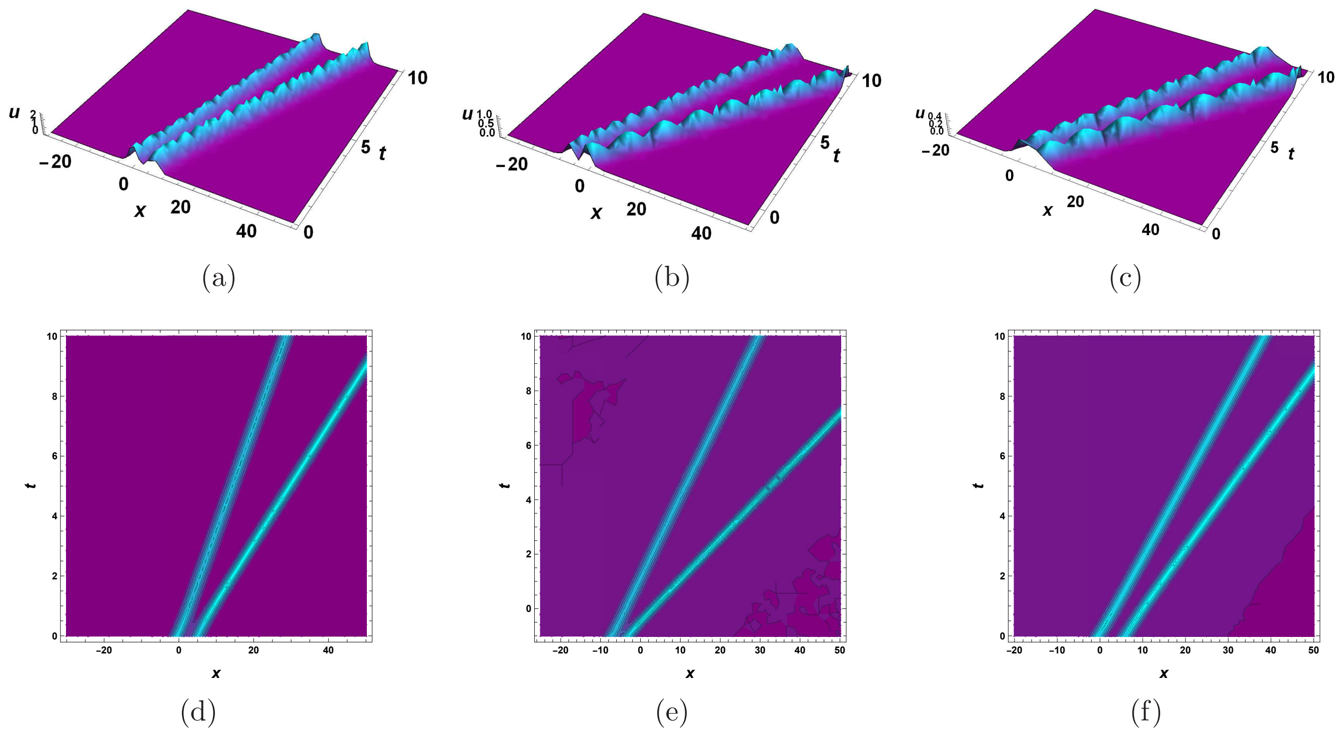

Figure 2: Two-soliton solutions via (12) for the function (17) with values: **(a)** For SK equation: $\kappa_1 = 1.35, \kappa_2 = 1.2, a = \frac{5}{3}, b = 5, c = 0$; **(b)** For Lax equation: $\kappa_1 = 1.5, \kappa_2 = 1.35, a = 10, b = 10, c = 5$; and **(c)** For CDG equation: $\kappa_1 = 1.4, \kappa_2 = 1.5, a = 60, b = 30, c = 0$. **(d-f)** portrays contour plots for **(a-c)** in xt-plane.

4.3. Three-soliton solution

To get three-soliton solution [34, 37], we consider the function Γ as

$$\Gamma(x,t) = 1 + e^{\Phi_1} + e^{\Phi_2} + e^{\Phi_3} + h_{12}e^{\Phi_1+\Phi_2} + h_{13}e^{\Phi_1+\Phi_3} + h_{23}e^{\Phi_2+\Phi_3} + g_{123}e^{\Phi_1+\Phi_2+\Phi_3}, \tag{24}$$

where h_{ij} with $1 \leq i < j \leq 3$ fulfills the relation (21) and dispersion coefficient g_{123} can be legitimized making use of symbolic computation to favour the following constraint:

$$g_{123} = h_{12}h_{13}h_{23}. \tag{25}$$

Therefore, from equation (24), we have

$$\Gamma_x = \sum_{m=1}^{3} \kappa_m e^{\Phi_m} + \sum_{1\leq m<n\leq 3} (\kappa_m+\kappa_n)h_{mn}e^{\Phi_m+\Phi_n} + \left(\sum_{m=1}^{3}\kappa_m\right) g_{123}e^{\Phi_1+\Phi_2+\Phi_3}, \tag{26}$$

$$\Gamma_{xx} = \sum_{m=1}^{3} \kappa_m^2 e^{\Phi_m} + \sum_{1\leq m<n\leq 3} (\kappa_m + \kappa_n)^2 h_{mn} e^{\Phi_m+\Phi_n} + \left(\sum_{m=1}^{3} \kappa_m \right)^2 g_{123} e^{\Phi_1+\Phi_2+\Phi_3}. \tag{27}$$

By substituting the equations (24), (26) and (27) into equation(12), we establish the three-soliton solution.

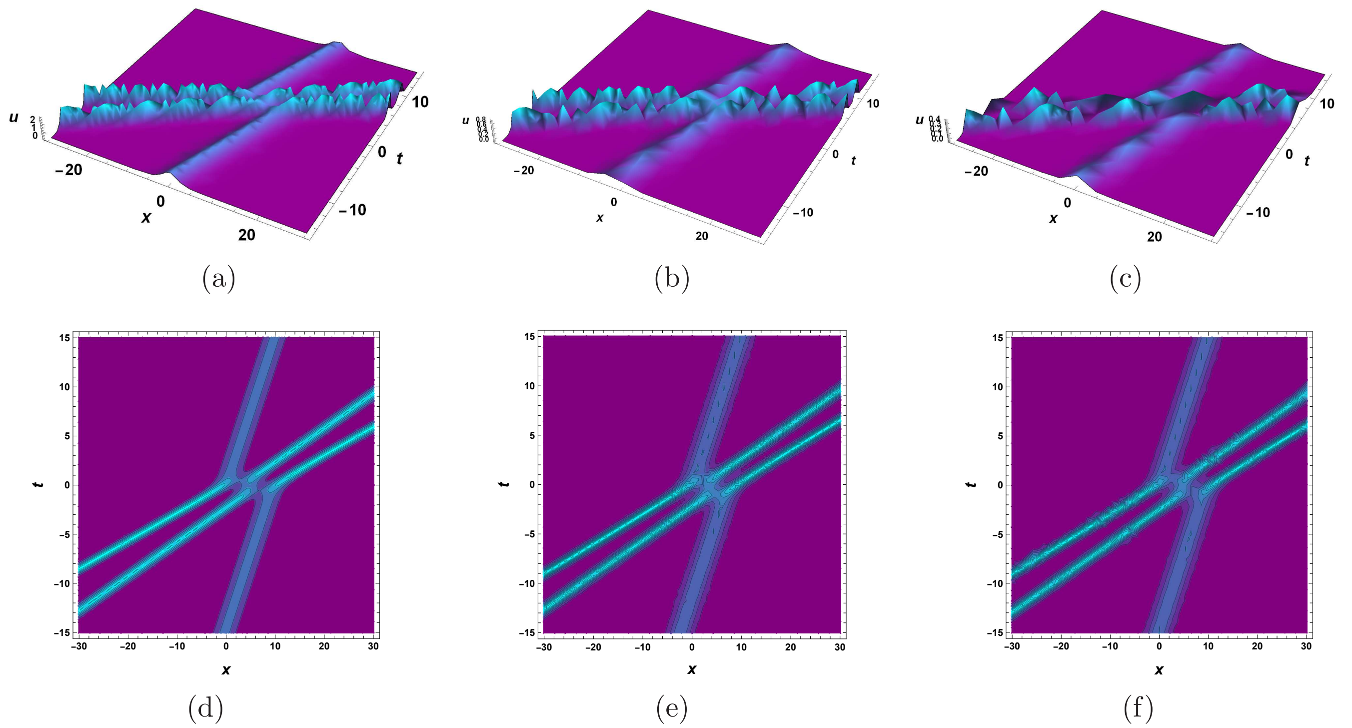

Figure 3: Three-soliton solutions via (12) for the function (24) with values: $\kappa_1 = 0.9, \kappa_2 = 1.3, \kappa_3 = 1.37$; **(a)** For SK equation: $a = \frac{5}{3}, b = 5, c = 0$; **(b)** For Lax equation: $a = 10, b = 10, c = 5$; and **(c)** For CDG equation: $a = 60, b = 30, c = 0$. **(d-f)** portrays contour plots for **(a-c)** in xt-plane.

In figure 1, the movable singularity forms the single-soliton wave profiles of the solution u, deduced by the equation (16), which is presented graphically for the function (13) with values $\kappa_1 = 0.25, a = \frac{5}{3}, b = 5, c = 0$; $\kappa_1 = 0.5, a = 10, b = 10, c = 5$; and $\kappa_1 = 0.35, a = 60, b = 30, c = 0$ for SK, Lax, and CDG equations, respectively. The singularity for the solution (16) can be obtained at $x = \frac{\log(-e^{tk_1^5})}{k_1}$ for any $k_1 \in R$ and $t \in R^+$. Moreover, the dependent variable u has a two-order singularity with respect to x. Similarly, we can observe the movable singularities for the figures 2, and 3 which show the two-soliton and three-soliton wave profiles respectively, for the resultant solution of the dependent variable u as suggested in the formulations with the appropriate choices of values for the parameters. The solution u for two-soliton and three-soliton is not shown due to its lengthy expression. However, we can analyze that the movable singularity occurs in $-20 \leq x \leq 20$ at $t = 0$ for the figures 2, and 3.

5. Conclusions

In this work, we proposed a fifth-order nonlinear evolution equation that is a generalization of the SawadaKotera, Lax, and CaudreyDoddGibbon equations to model the ion-acoustic waves in plasma physics, the nonlinear waves in shallow water, vibrations in mechanical engineering and other nonlinear fields. Painlevé analysis was used to test the integrability of the constructed equation and validate its dependence on the constant parameters in the equation. We performed a general logarithmic transformation for the dependent variable on the established equation by solving a system of equations constructed from linear combinations of given values and assumed variables. Multiple soliton solutions have been produced using the simplified Hirota method, and the dynamical behavior of resultant soliton solutions has been presented.

Conflict of Interest

The authors declare that they have no conflict of interest.

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