A direct symbolic computation of center-controlled rogue waves to a new Painlevé integrable (3+1)-D generalized nonlinear evolution equation in plasmas

Sachin Kumar* **Brij Mohan**†

*Department of Mathematics, Faculty of Mathematical Sciences, University of Delhi, Delhi 110007, India

†Department of Mathematics, Hansraj College, University of Delhi, Delhi -110007, India

Abstract: This paper proposes a new integrable generalized (3+1)-dimensional nonlinear partial differential equation. We apply the standard Painlevé test to check the integrability, which shows the complete integrability of this equation. We employ symbolic computation directly to create the rogue waves using the center-controlled parameters β and γ. We create first, second, and third-order rogue wave solutions via direct computation for various values of center-controlled parameters and suitable choices of different constants in the said equation. We obtain the bilinear equation in the auxiliary function f of the transformed variables ξ and η by using the transformation for dependent variable u. Using Hirota's direct method to create rogue waves up to the third order, we apply the generalized formula for rogue waves formulated by N-soliton. Using the symbolic system tool *Mathematica*, we illustrate the dynamics for the rogue wave solutions with various center-controlled parameters. We demonstrate how massive rogue waves, present in many nonlinear events, behave dominantly over tiny rogue waves. The equation investigates the development of long waves with small amplitudes traveling in plasma physics and wave motion in fluids and other weakly dispersive mediums. Scientific areas, including oceanography, fluid dynamics, dusty plasma, optical fibers, nonlinear dynamics, and numerous other nonlinear fields, show the occurrence of rogue waves in one way or another.

Keywords: Logarithmic transformation, Bilinearization, Integrability analysis, Generalized nonlinear equation, Multi-order rogue waves.

1 Introduction

Nonlinear partial differential equations (PDEs) are a broad branch of mathematics and physics that deal with equations involving partial derivatives of nonlinear functions that reflect the models of several complicated physical procedures in various research and engineering sectors. Mathematicians have utilized nonlinear PDEs to answer issues like the Poincaré conjecture and the Calabi conjecture and describe various physical systems, from fluid dynamics to gravity. Nonlinear PDEs are challenging to examine since few universal methods apply to these equations. Usually, it is required to study each equation as a particular problem. Nevertheless, in some circumstances, general approaches can be used. For instance, numerical techniques can roughly solve nonlinear partial differential equations. These methods use finite differences or other approaches to approximate the solution at each point by discretizing the problem's domain into a grid of points. Several methods for locating analytical and exact solutions have been proposed to deal with nonlinear PDEs such as the simplified Hirota's technique [1–3], Hirota's bilinearization method [4–8], the

* *sachinambariya@gmail.com*

† *brijmohan6414@gmail.com (Corresponding author)*

Bäcklund transformation [9,10], the Darboux transformation [11–13], the Pfaffian technique [14,15], the Lie symmetry analysis [16–19], Inverse scattering method [20,21], Bilinear neural network method [22–24], and other techniques [25,26].

Rogue waves [27–34], often called giant solitary waves that appear out of nowhere in the ocean, are localized in space-time and have a sizable amplitude. They are unexpected and widespread and may do great harm to individuals. Exploring the evolving process of rogue waves is essential, and many academics are interested. Their excessively steep height distinguishes them, sometimes more significant than nearby waves. Nonlinear wave dynamics study rogue waves because they contradict conventional linear wave models. Nonlinear science research on rogue waves tries to comprehend their fundamental mechanics and forecast their occurrence. The rogue waves appear out of nowhere when smaller waves concentrate their energy in a narrow region. The improvement of marine safety is one important use. Scientists can give early detection and warning systems to stop mishaps brought on by rogue waves by creating models and prediction algorithms. The maritime sector, offshore oil and gas platforms, and coastal infrastructure may all benefit from this information. Therefore, better operational safety and cost-effective solutions can be achieved by comprehending their dynamics to build safer structures and create tactics to lessen their effects. Furthermore, studying their causes and dynamics contributes to our growing understanding of complex systems, nonlinear wave interactions, and the formation of extreme occurrences in various physical and mathematical situations.

Recently, in 2022, X. Yang, Z. Zhang, et al. [35] gave a direct method for constructing N-order rogue waves of the (3+1)-D KdV-Benjamin-Bona-Mahony equation from N-soliton by Hirota method using long wave limit technique. Utilizing the direct method by Hirota in soliton theory [4], they admitted the rogue wave solutions as

$$f_N = \sum_{\zeta=0,1} \exp\left(\sum_{i=1}^{N} \zeta_i \eta_i + \sum_{1=i<j}^{N} A_{ij}\zeta_i\zeta_j\right),$$

where $\zeta_i = 0, 1$ for $1 \leq i \leq N$, $\sum_{\zeta=0,1}$ as the summation of all possible combinations ζ, η_i are the phase variables and $\exp(A_{ij})$ are the constants. In their work, they generated first, second and third-order rogue wave solutions for f_2, f_4 and f_6 respectively. If we follow the pattern of the their solutions then we can generalized the N-rogue wave solution as

$$f_N = \sum_{k=0}^{\frac{N(N+1)}{2}} \sum_{i=0}^{k} a_{N(N+1)-2k,2i}(\eta)^{2i}(\xi)^{N(N+1)-2k},$$

with transformed variables ξ and η, which gives resemblance to the function used in symbolic computational approach [36–41] given by Zhaqilao [42]. This approach offers a straightforward method for locating N-order rogue waves of the nonlinear PDEs, which includes a phase in which the equation is first transformed into a Hirota's bilinear form, using a dependent variable transformation.

This article investigates a new (3+1)-dimensional generalized nonlinear evolution equation

$$u_{xxxy} + \mu_1 u_{xt} + \mu_2 u_{yt} + \mu_3(u_x u_y)_x + \mu_4 u_{xx} + \mu_5 u_{zz} = 0, \tag{1}$$

where the coefficients $\mu_i; 1 \leq i \leq 5$ are real parameters. The equation (1) generalizes the well known equations as

- For $\mu_1 = 0, \mu_2 = -1, \mu_3 = \mu_4 = 3, \mu_5 = -3$, equation (1) gives (3+1)-D Hirota bilinear equation [36] as
$$u_{xxxy} - u_{yt} + 3(u_x u_y)_x + 3u_{xx} - 3u_{zz} = 0. \tag{2}$$
- For $\mu_1 = \mu_2 = 1, \mu_3 = 3, \mu_4 = 0, \mu_5 = -1$, equation (1) becomes (3+1)-D KP equation [39] as
$$u_{xxxy} + u_{xt} + u_{yt} + 3(u_x u_y)_x - u_{zz} = 0. \tag{3}$$

It is a known fact that all integrable nonlinear PDEs carry exponentially localized solutions in specific directions, including solitons [43–47], lumps [48], optical solitons [49, 50], and other solutions. Finding exact and analytical solutions can be achieved by looking at the integrability of nonlinear PDEs. The Painlevé test can verify complete integrability for a nonlinear PDE [51, 52]. Determining whether a PDE passes the Painlevé test becomes highly tedious, but symbolic tools including *Mathematica*, *Matlab*, and others software makes it possible to carry out this analysis. We look for particular answers to accurately understand the characteristics of various facts in various fields of natural sciences. Because nonlinear PDE resembles real-world circumstances and can provide a wide variety of solutions, it has drawn the attention of several researchers, as was already mentioned. Software from the Computer Algebra System (CAS)can assist in finding such solutions. Using Mathematica, Baldwin and Hereman [53] proposed a symbolic computation in 2006 that can carry out the Painlevé test for PDEs, which is built using the WTC-Krushkal approach [54]. The dynamical study of rogue waves behavior produced for nonlinear PDEs has been an exciting field of study to illustrate fundamental aspects of engineering sciences, dusty plasma, oceanography, complex physical systems, shallow water waves and several nonlinear fields.
Here is the paper's structure: The following section examines integrability of the considered equation employing Painlevé test. Section 3 describes the direct symbolic computational used to study the said equation. In section 4, Firstly, we apply the dimensionless transformation to the dependent variable $u(x, y, z, t)$ to change the equation in $u(\xi, \eta)$, then find the dispersion relation with the phase variable and transform the changed equation into a bilinear equation using logarithmic transformation. Section 5 obtains the center-controlled first, second, and third-order rogue waves and depict the dynamics for the same with different center parameters. In section 6, we will go over the results of the obtained solutions and their related graphics. The final section summarizes the findings and conclusions of the investigation.

2 Analysis of Painlevé test

To test the integrability of the Eq. (1), we apply the Painlevé test given by Weiss, Tabor and Carnevale (WTC) [54], which is widely used among physicists and mathematicians to check the integrability by verifying the integrable conditions for a given PDE. This test goes thorugh three steps, first it alalyzes the leading order behavior, then find the points of resonance, and lastly verify the conditions for resonance points. In this test, if all the movable singularities corrsponding to the solutions are simple poles, then the system is said to be Painlevé integrable.The field u in the Eq. (1) is expanded by a Laurent's series with g, an analytic function of $\{x, y, z, t\}$, about the singularity manifold $g(x, y, z, t) = 0$ as

$$u = \sum_{j=0}^{\infty} u_j g^{j+\lambda}, \tag{4}$$

where λ is an integer and $u_j = u_j(x, y, z, t); j = 0, 1, 2, ...;$ are the arbitrary functions. On substituting the Eq. (4) into Eq. (1), we calculate λ by resembling the dominance terms with leading order analysis as

$$\lambda = -1.$$

Also, we encounter the dominant behavior concerning the resonance j as

$$u_0 = \frac{6g_x}{\mu_3},$$

and characteristic relation for the resonances is

$$(-6 + r)(-4 + r)(-1 + r)(1 + r)\mu_3 g_y g_x^3 = 0. \tag{5}$$

Thus we get the resonances as

$$j = -1, \ \ 1, \ \ 4, \ \ 6.$$

The irrational choice for singularity manifold $\rho(x, y, z, t) = 0$ gives the resonance $j = -1$. The expressions u_j exist explicitly for $j = 0, 2, 3, 5$ and arbitrary choices for $j = 1, 4, 6$, are given as

$$\begin{aligned}
u_1 &= u_1(x, y, z, t), \\
u_2 &= -\frac{\mu_3 g_{\mathrm{x}}^2 (u_1)_{\mathrm{y}} + \mu_3 g_{\mathrm{x}} g_{\mathrm{y}} (u_1)_{\mathrm{x}} + \mu_1 g_{\mathrm{t}} g_{\mathrm{x}} + \mu_2 g_{\mathrm{t}} g_{\mathrm{y}} + \mu_4 g_{\mathrm{x}}^2 + 3 g_{\mathrm{x}} g_{\mathrm{xxy}} - 3 g_{\mathrm{xx}} g_{\mathrm{xy}} + g_{\mathrm{xxx}} g_{\mathrm{y}} + \mu_5 g_{\mathrm{z}}^2}{2\mu_3 g_{\mathrm{x}}^2 g_{\mathrm{y}}}, \\
u_3 &= \frac{\mu_3 g_{\mathrm{x}}^2 g_{\mathrm{y}}^3 (u_1)_{\mathrm{xx}} + \mu_3 g_{\mathrm{x}}^2 g_{\mathrm{xx}} g_{\mathrm{y}}^2 (u_1)_{\mathrm{y}} - 2\mu_3 g_{\mathrm{x}} g_{\mathrm{xx}} g_{\mathrm{y}}^3 (u_1)_{\mathrm{x}} + \cdots + 9 g_{\mathrm{xx}}^2 g_{\mathrm{xy}} g_{\mathrm{y}}^2 - 3\mu_5 g_{\mathrm{xx}} g_{\mathrm{y}}^2 g_{\mathrm{z}}^2}{8\mu_3 g_{\mathrm{x}}^4 g_{\mathrm{y}}^3}, \\
u_4 &= u_4(x, y, z, t), \\
u_5 &= -\frac{g_{\mathrm{y}} g_{\mathrm{yy}} \mu_4^2 g_{\mathrm{x}}^7 + g_{\mathrm{y}} g_{\mathrm{yy}} \mu_3^2 (u_1)_{\mathrm{y}}^2 g_{\mathrm{x}}^7 + 2 g_{\mathrm{y}} g_{\mathrm{yy}} \mu_3 \mu_4 (u_1)_{\mathrm{y}} g_{\mathrm{x}}^7 + \cdots - 120 g_{\mathrm{xx}}^3 g_{\mathrm{y}}^4 g_{\mathrm{z}}^2 \mu_5}{96 g_{\mathrm{x}}^8 g_{\mathrm{y}}^5 \mu_3}, \\
u_6 &= u_6(x, y, z, t).
\end{aligned}$$

The terms in u_3 and u_5 has been skipped due to very lengthy expressions. The resonances j satisfy the condition for compatibility identically. Furthermore, it exhibits that the concerned Eq. (1) is completely Painlevé integrable.

3 Description of direct symbolic computation

Let us consider a (3+1)-dimensional nonlinear PDE as

$$P(u, u_t, u_{xt}, u_{yt}, u_{zt}, u_x, u_{xx}, u_{xy}, u_{xz}, \cdots) = 0, \tag{6}$$

where subscripts represent partial derivatives concerning the independent variables x, y, z, and t.
Consider a transformation as

$$u = u(\xi, \eta), \qquad \xi(x, t) = c_1 x + c_2 t, \quad \eta(y, z) = c_3 y + c_4 z, \tag{7}$$

with constants $c_i; 1 \leq i \leq 4$ to the nonlinear PDE (6).
On applying the transformation (7) into the Eq. (6), we get a PDE as

$$Q(u, u_\xi, u_\eta, u_{\xi\eta}, u_{\xi\xi}, u_{\eta\eta}, \cdots) = 0, \tag{8}$$

Next, we consider a dependent variable transformation as

$$u(\xi, \eta) = R(\ln f)_{\xi^n}, \tag{9}$$

where R is a non-zero parameter to be determined later, $f = f(\xi, \eta)$ is a auxiliary function of dependent variables ξ and η, where n is the order of partial differentiation w.r.t. ξ depending upon the balance of the term of higher-order and nonlinear term in Eq. (8).
We assume the auxiliary function f, a generalized form of rogue wave solutions [35] governed by N-solitons Hirota bilinear approach as

$$f(\xi, \eta) = \widehat{f_n}(\xi, \eta, \beta, \gamma) = \sum_{k=0}^{\frac{n(n+1)}{2}} \sum_{i=0}^{k} a_{n(n+1)-2k, 2i} (\eta - \gamma)^{2i} (\xi - \beta)^{n(n+1)-2k}, \tag{10}$$

where $a_{l,m}; l, m \in \{0, 2, \cdots, k(k+1)\}$ are constants to be determined later and β, γ are the real parameters that control of the wave center.

4 Logarithmic transformation and bilinear equation

By considering $u = u(\xi, \eta)$ with $\xi = x + t$ and $\eta = y + z$ in equation (1), we get

$$u_{\xi\xi\xi\eta} + \mu_1 u_{\xi\xi} + \mu_2 u_{\xi\eta} + \mu_3 \left(u_\eta u_{\xi\xi} + u_\xi u_{\xi\eta}\right) + \mu_4 u_{\xi\xi} + \mu_5 u_{\eta\eta} = 0. \tag{11}$$

By taking the phase variable Φ_i in the equation (11) as

$$\Phi_i = p_i \xi - w_i \eta, \tag{12}$$

where p_i are the real parameters, and w_i for $i = 1, 2, 3, ...,$ are the dispersion coefficients. By putting $u(\xi, \eta) = e^{\Phi_i}$ in terms with linearity of Eq. (11), we get

$$w_i = \frac{p_i^3 + \mu_2 p_i + \pm p_i \sqrt{2\mu_2 p_i^2 + p_i^4 + \mu_2^2 - 4(\mu_1 + \mu_4)\mu_5}}{2\mu_5}. \tag{13}$$

Now, we assume the logarithmic transformation

$$u(\xi, \eta) = R(\ln f)_\xi, \tag{14}$$

and substitute this transformation with equation (13) and $f(\xi, \eta) = 1 + e^{\Phi_1}$ in equation (11). On solving for R, we find

$$R = \frac{6}{\mu_3}.$$

Thus, the dependent variable transformation (14) becomes

$$u(\xi, \eta) = \frac{6}{\mu_3}(lnf)_\xi. \tag{15}$$

Substitution of transformation (15) into equation (11) converts to the bilinear equation in $f(\xi, \eta)$ as

$$f_\xi \left(\mu_2 f_\eta + 3f_{\xi\xi\eta}\right) + f_\eta(\mu_5 f_\eta + f_{\xi\xi\xi}) - f_{\xi\eta}\left(\mu_2 f + 3f_{\xi\xi}\right) - (\mu_1 + \mu_4)(f f_{\xi\xi} - f_\xi^2) - f(\mu_5 f_{\eta\eta} + f_{\xi\xi\xi\eta}) = 0. \tag{16}$$

5 Rogue wave solutions with center-controlled parameters

5.1 Rogue waves of first-order

For a first-order rogue wave, we choose dependent variable function $f(\xi, \eta)$ for $n = 1$ in equation (10) as

$$f(\xi, \eta) = a_{0,0} + a_{0,2}\eta^2 + a_{2,0}\xi^2. \tag{17}$$

On substituting Eq. (17) into the Eq. (16), and equating coefficients of distinct powers of $\xi^r \eta^s; r, s \in Z$ to zero, we get a system as

$$\begin{aligned} \mu_1 a_{0,0} a_{2,0} + \mu_4 a_{0,0} a_{2,0} + \mu_5 a_{0,0} a_{0,2} &= 0, \\ \mu_1 a_{2,0}^2 + \mu_4 a_{2,0}^2 - \mu_5 a_{0,2} a_{2,0} &= 0, \\ \mu_1 a_{2,0} a_{0,2} + \mu_4 a_{2,0} a_{0,2} - \mu_5 a_{0,2}^2 &= 0. \end{aligned} \tag{18}$$

On solving this system , we obtain the constants as

$$a_{0,0} = 0, \quad a_{0,2} = \frac{\left(\mu_1 + \mu_4\right) a_{2,0}}{\mu_5}, \quad a_{2,0} = a_{2,0}. \tag{19}$$

Thus, the equation (17) with values in (19) becomes

$$f(\xi,\eta) = \widehat{F_1}(\xi,\eta,\beta,\gamma) = a_{2,0}\left((\xi-\beta)^2 + \frac{(\mu_1+\mu_4)(\eta-\gamma)^2}{\mu_5}\right), \tag{20}$$

which is a solution of equation (16) with center controlled parameters β and γ. By substituting equation (20) into (15), we get a rogue wave solution as

$$u(\xi,\eta) = \frac{12\mu_5(\xi-\beta)}{\mu_3\left(\mu_5(\xi-\beta)^2 + (\mu_1+\mu_4)(\gamma-\eta)^2\right)}, \tag{21}$$

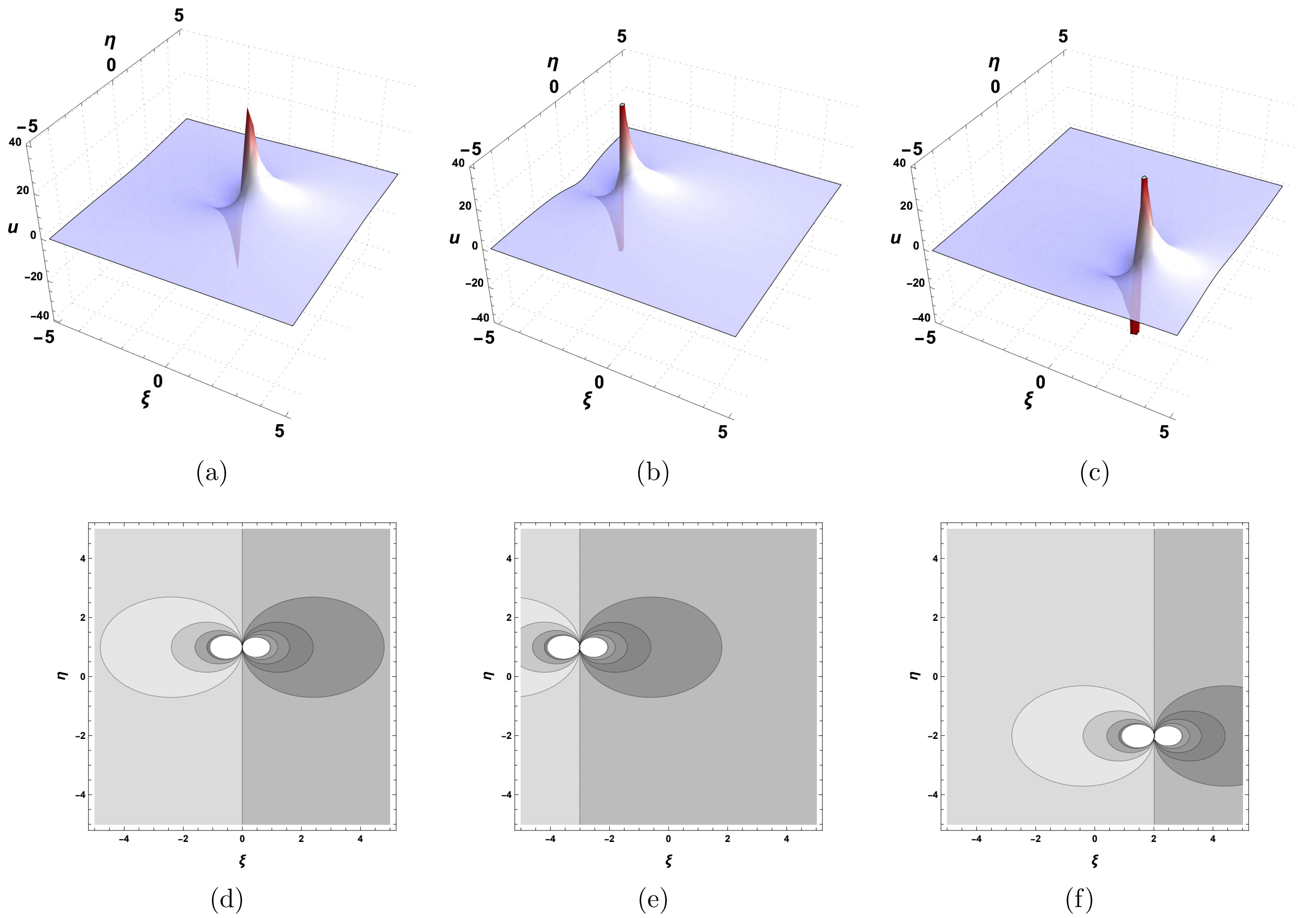

Figure 1: Rogue waves of first-order for (21) with (20) having values: $\mu_1 = \mu_3 = \mu_4 = \mu_5 = 1$, and center-controlled parameters as: **(a)** $\beta = 0, \gamma = 0$; **(b)** $\beta = -3, \gamma = 1$; and **(c)** $\beta = 2, \gamma = -2$. **(d-f)** are 2-D contours for **(a-c)** w.r.t $\xi\eta$-plane.

5.2 Rogue waves of second-order

To get the rogue wave of second-order, we choose dependent variable function $f(\xi,\eta)$ for $n = 2$ in equation (10) as

$$f(\xi,\eta) = a_{0,0} + a_{0,2}\eta^2 + a_{0,4}\eta^4 + a_{0,6}\eta^6 + a_{2,0}\xi^2 + a_{2,2}\xi^2\eta^2 + a_{2,4}\xi^2\eta^4 + a_{4,0}\xi^4 + a_{4,2}\xi^4\eta^2 + a_{6,0}\xi^6. \tag{22}$$

Substituting Eq. (22) into the bilinear Eq. (16), and equating coefficients of distinct powers of $\xi^r\eta^s; r, s \in Z$ to zero, gives a system. On solving this system of equations, we compute the constants as

$$a_{0,0} = \frac{29\mu_5 a_{4,2}}{2\mu_2^3(\mu_1+\mu_4)}, \quad a_{0,2} = \frac{231 a_{4,2}}{4\mu_2^2}, \quad a_{0,4} = \frac{5(\mu_1+\mu_4)a_{4,2}}{\mu_2\mu_5},$$

$$a_{0,6} = \frac{(\mu_1+\mu_4)^2 a_{4,2}}{3\mu_5^2}, \quad a_{2,0} = -\frac{9\mu_5 a_{4,2}}{4\mu_2^2(\mu_1+\mu_4)}, \quad a_{2,2} = \frac{12 a_{4,2}}{\mu_2},$$

$$a_{2,4} = \frac{(\mu_1+\mu_4)a_{4,2}}{\mu_5}, \quad a_{4,0} = -\frac{\mu_5 a_{4,2}}{\mu_2(\mu_1+\mu_4)},$$

$$a_{6,0} = \frac{\mu_5 a_{4,2}}{3(\mu_1+\mu_4)}, \tag{23}$$

where $a_{4,2}$ is an arbitrary constant. Thus, the equation (17) with values in (23) becomes

$$\begin{aligned} f(\xi,\eta) = \widehat{F_2}(\xi,\eta,\beta,\gamma) = \frac{a_{4,2}}{12}\Big(&\frac{12(\mu_1+\mu_4)(\xi-\beta)^2(\eta-\gamma)^4}{\mu_5} + \frac{144(\xi-\beta)^2(\eta-\gamma)^2}{\mu_2} + 12(\xi-\beta)^4(\eta-\gamma)^2 \\ &+ \frac{4\mu_5(\xi-\beta)^6}{\mu_1+\mu_4} - \frac{12\mu_5(\xi-\beta)^4}{\mu_2(\mu_1+\mu_4)} - \frac{27\mu_5(\xi-\beta)^2}{\mu_2^2(\mu_1+\mu_4)} + \frac{4(\mu_1+\mu_4)^2(\eta-\gamma)^6}{\mu_5^2} \\ &+ \frac{60(\mu_1+\mu_4)(\eta-\gamma)^4}{\mu_2\mu_5} + \frac{693(\eta-\gamma)^2}{\mu_2^2} + \frac{174\mu_5}{\mu_2^3(\mu_1+\mu_4)}\Big), \end{aligned} \tag{24}$$

which is a solution of equation (16) with center controlled parameters β and γ. By substituting equation (24) into (15), we get a second-order rogue wave solution as

$$u(\xi,\eta) = \frac{6}{\mu_3}(\ln \widehat{F_2}(\xi,\eta,\beta,\gamma))_\xi. \tag{25}$$

5.3 Rogue waves of third-order

For a rogue wave of third-order, we take dependent variable function $f(\xi,\eta)$ for $n = 3$ in equation (10) as

$$\begin{aligned} f(\xi,\eta) = {}& a_{0,0} + a_{0,2}\eta^2 + a_{0,4}\eta^4 + a_{0,6}\eta^6 + a_{0,8}\eta^8 + a_{0,10}\eta^{10} + a_{0,12}\eta^{12} + a_{2,0}\xi^2 + a_{2,2}\eta^2\xi^2 + a_{2,4}\xi^2\eta^4 \\ &+ a_{2,6}\xi^2\eta^6 + a_{2,8}\xi^2\eta^8 + a_{2,10}\xi^2\eta^{10} + a_{4,0}\xi^4 + a_{4,2}\xi^4\eta^2 + a_{4,4}\eta^4\xi^4 + a_{4,6}\xi^4\eta^6 + a_{4,8}\xi^4\eta^8 + a_{6,0}\xi^6 \\ &+ a_{6,2}\xi^6\eta^2 + a_{6,4}\xi^6\eta^4 + a_{6,6}\xi^6\eta^6 + a_{8,0}\xi^8 + a_{8,2}\xi^8\eta^2 + a_{8,4}\xi^8\eta^4 + a_{10,0}\xi^{10} + a_{10,2}\xi^{10}\eta^2 + a_{12,0}\xi^{12}. \end{aligned} \tag{26}$$

Substituting Eq. (26) into the bilinear Eq. (16), and equating coefficients of distinct powers of $\xi^r\eta^s; r, s \in Z$ to zero, gives a system. By solving the system of equations, we compute the constants as

$$a_{0,0} = \frac{7353680000\mu_5 a_{10,2}}{1113\mu_2^6(\mu_1+\mu_4)}, \quad a_{0,2} = -\frac{6077833600 a_{10,2}}{371\mu_2^5}, \quad a_{0,4} = -\frac{6359200(\mu_1+\mu_4)a_{10,2}}{7\mu_2^4\mu_5},$$

$$a_{0,6} = -\frac{800(\mu_1+\mu_4)^2 a_{10,2}}{3\mu_2^3\mu_5^2}, \quad a_{0,8} = \frac{160(\mu_1+\mu_4)^3 a_{10,2}}{\mu_2^2\mu_5^3}, \quad a_{0,10} = -\frac{10(\mu_1+\mu_4)^4 a_{10,2}}{\mu_2\mu_5^4},$$

$$a_{0,12} = \frac{(\mu_1+\mu_4)^5 a_{10,2}}{6\mu_5^5}, \quad a_{2,0} = \frac{72534400\mu_5 a_{10,2}}{371\mu_2^5(\mu_1+\mu_4)}, \quad a_{2,2} = \frac{1521600 a_{10,2}}{7\mu_2^4}, \quad a_{2,4} = -\frac{800(\mu_1+\mu_4)a_{10,2}}{\mu_2^3\mu_5},$$

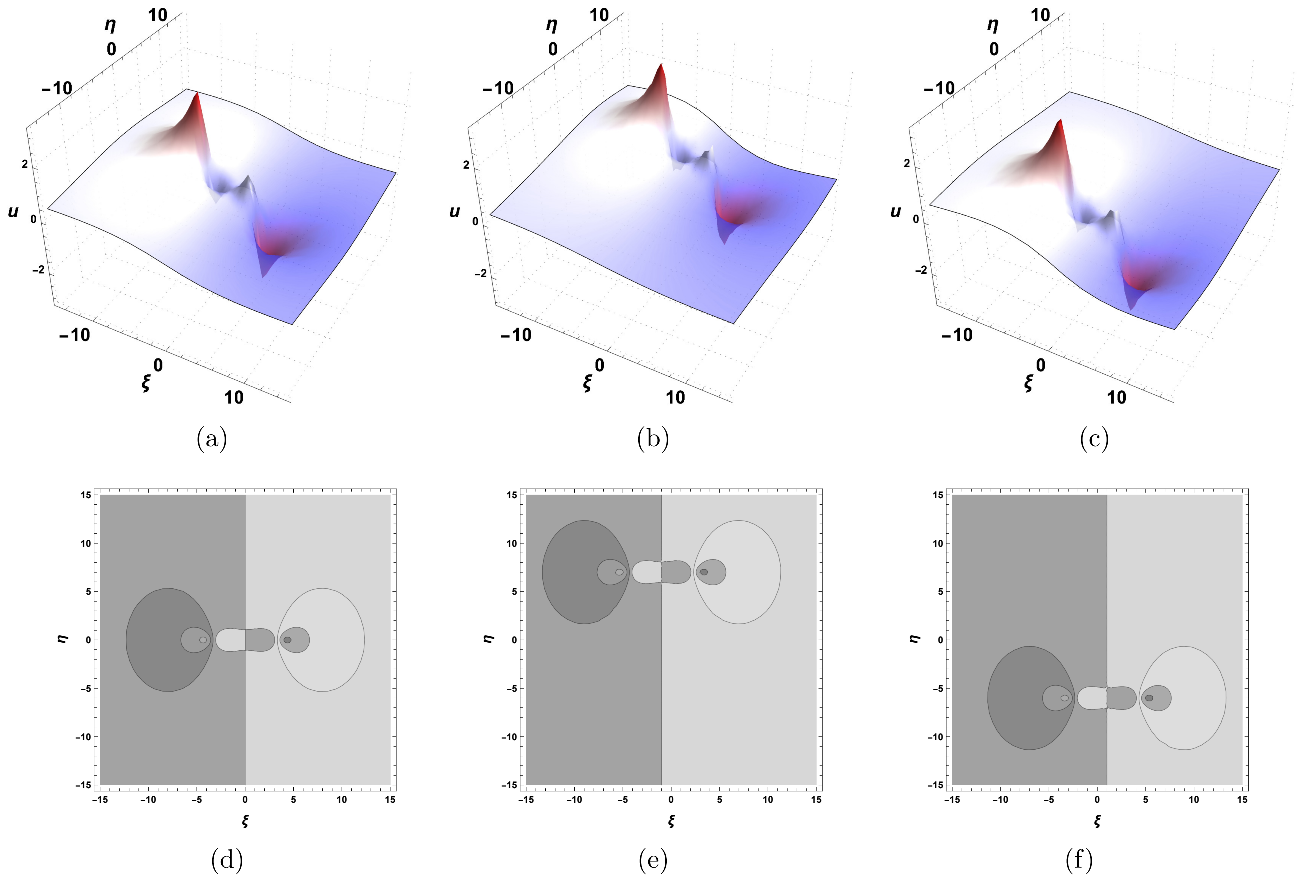

Figure 2: Rogue waves of second-order for (25) with (24) having values: $\mu_1 = 5, \mu_2 = 0.3, \mu_3 = -3, \mu_4 = 0, \mu_5 = 9$, and center-controlled parameters as: **(a)** $\beta = 0, \gamma = 0$; **(b)** $\beta = -1, \gamma = 7$; and **(c)** $\beta = 1, \gamma = -6$. **(d-f)** are 2-D contours for **(a-c)** w.r.t $\xi\eta$-plane.

$$
\begin{gathered}
a_{2,6} = \frac{560\left(\mu_1 + \mu_4\right)^2 a_{10,2}}{\mu_2^2 \mu_5^2}, \qquad a_{2,8} = -\frac{30\left(\mu_1 + \mu_4\right)^3 a_{10,2}}{\mu_2 \mu_5^3}, \qquad a_{2,10} = \frac{\left(\mu_1 + \mu_4\right)^4 a_{10,2}}{\mu_5^4}, \\
a_{4,0} = -\frac{55200 \mu_5 a_{10,2}}{7 \mu_2^4 \left(\mu_1 + \mu_4\right)}, \qquad a_{4,2} = \frac{800 a_{10,2}}{\mu_2^3}, \qquad a_{4,4} = \frac{800\left(\mu_1 + \mu_4\right) a_{10,2}}{\mu_2^2 \mu_5} \\
a_{4,6} = -\frac{20\left(\mu_1 + \mu_4\right)^2 a_{10,2}}{\mu_2 \mu_5^2}, \qquad a_{4,8} = \frac{5\left(\mu_1 + \mu_4\right)^3 a_{10,2}}{2 \mu_5^3}, \qquad a_{6,0} = \frac{800 \mu_5 a_{10,2}}{3 \mu_2^3 \left(\mu_1 + \mu_4\right)}, \\
a_{6,2} = \frac{560 a_{10,2}}{\mu_2^2}, \qquad a_{6,4} = \frac{20\left(\mu_1 + \mu_4\right) a_{10,2}}{\mu_2 \mu_5}, \qquad a_{6,6} = \frac{10\left(\mu_1 + \mu_4\right)^2 a_{10,2}}{3 \mu_5^2}, \\
a_{8,0} = \frac{160 \mu_5 a_{10,2}}{\mu_2^2 \left(\mu_1 + \mu_4\right)}, \qquad a_{8,2} = \frac{30 a_{10,2}}{\mu_2}, \qquad a_{8,4} = \frac{5\left(\mu_1 + \mu_4\right) a_{10,2}}{2 \mu_5}, \qquad a_{10,0} = \frac{10 \mu_5 a_{10,2}}{\mu_2 \left(\mu_1 + \mu_4\right)} \\
a_{12,0} = \frac{\mu_5 a_{10,2}}{6\left(\mu_1 + \mu_4\right)},
\end{gathered}
\tag{27}
$$

where $a_{10,2}$ is an arbitrary constant. Thus, the equation (17) with values in (27) becomes

$$\begin{aligned}
f(\xi,\eta) = \widehat{F_3}(\xi,\eta,\beta,\gamma) = \frac{a_{10,2}}{2226}\Big(&\frac{371A^5C^{12}}{\mu_5^5} + \frac{2226A^4B^2C^{10}}{\mu_5^4} - \frac{22260A^4C^{10}}{\mu_2\mu_5^4} + \frac{5565A^3B^4C^8}{\mu_5^3} \\
&- \frac{66780A^3B^2C^8}{\mu_2\mu_5^3} + \frac{356160A^3C^8}{\mu_2^2\mu_5^3} + \frac{7420A^2B^6C^6}{\mu_5^2} - \frac{44520A^2B^4C^6}{\mu_2\mu_5^2} + \frac{1246560A^2B^2C^6}{\mu_2^2\mu_5^2} \\
&- \frac{593600A^2C^6}{\mu_2^3\mu_5^2} + \frac{371B^{12}\mu_5}{A} + \frac{22260B^{10}\mu_5}{A\mu_2} + \frac{5565AB^8C^4}{\mu_5} + \frac{356160B^8\mu_5}{A\mu_2^2} + \frac{44520AB^6C^4}{\mu_2\mu_5} \\
&+ \frac{593600B^6\mu_5}{A\mu_2^3} + \frac{1780800AB^4C^4}{\mu_2^2\mu_5} - \frac{17553600B^4\mu_5}{A\mu_2^4} - \frac{1780800AB^2C^4}{\mu_2^3\mu_5} + \frac{435206400B^2\mu_5}{A\mu_2^5} \\
&- \frac{2022225600AC^4}{\mu_2^4\mu_5} + \frac{14707360000\mu_5}{A\mu_2^6} + 2226B^{10}C^2 + \frac{66780B^8C^2}{\mu_2} + \frac{1246560B^6C^2}{\mu_2^2} \\
&+ \frac{1780800B^4C^2}{\mu_2^3} + \frac{483868800B^2C^2}{\mu_2^4} - \frac{36467001600C^2}{\mu_2^5}\Big),
\end{aligned} \tag{28}$$

where $A = (\mu_1 + \mu_4), B = (\xi - \beta)$ and $C = (\eta - \gamma)$, which is a solution of equation (16) with center controlled parameters β and γ. By substituting equation (28) into (15), we obtain a third-order rogue wave as

$$u(\xi,\eta) = \frac{6}{\mu_3}(\ln \widehat{F_3}(\xi,\eta,\beta,\gamma))_\xi. \tag{29}$$

6 Results and discussion

The novel nonlinear PDE equation (1) can have a variety of solutions, including solitons, rogue waves, lumps, and others, due to its perfect integrability. With the proper parameter selections, we could find the first, second, and third-order center-controlled rogue waves using the discussed symbolic computation and demonstrate the dynamics of the solutions. In addition, the explanation of the findings is as follows:

- Figure 1 showcase the rogue waves of first-order with center-controlled parameters due to the singularity around $\xi = \beta$ with the parameters $\mu_1 = \mu_3 = \mu_4 = \mu_5 = 1$, and center parameters as $\beta = 0, \gamma = 0$; $\beta = -3, \gamma = 1$; and $\beta = 2, \gamma = -2$, for (a), (b), and (c), respectively.

- In figure 2, we depict the rogue waves of second-order with center-controlled parameters (β, γ). Figure shows two rogue waves are generated due to the singularity where the large rogue dominates the other rogue wave, with the parameters $\mu_1 = 5, \mu_2 = 0.3, \mu_3 = -3, \mu_4 = 0, \mu_5 = 9$, and center parameters as $\beta = 0, \gamma = 0$; $\beta = -1, \gamma = 7$; and $\beta = 1, \gamma = -6$, for (a), (b), and (c), respectively.

- Figure 3 illustrate the third-order rogue waves with center-controlled parameters (β, γ). It shows that rogue waves parallel to ξ-axis dominates the rogue waves parallel to η-axis with the parameters $\mu_1 = 4.5, \mu_2 = -3.5, \mu_3 = 10, \mu_4 = -3, \mu_5 = 10$ and center parameters $\beta = 0, \gamma = 0$; $\beta = -5, \gamma = 3$; and $\beta = 5, \gamma = -3$ for (a), (b), and (c), respectively.

7 Conclusions

This paper has investigated a new integrable generalized (3+1)-dimensional nonlinear evolution equation. To test the integrability of the concerned PDE, we performed the Painlevé test that gave the complete integrability of this equation. We used symbolic computation directly to generate the rogue waves with

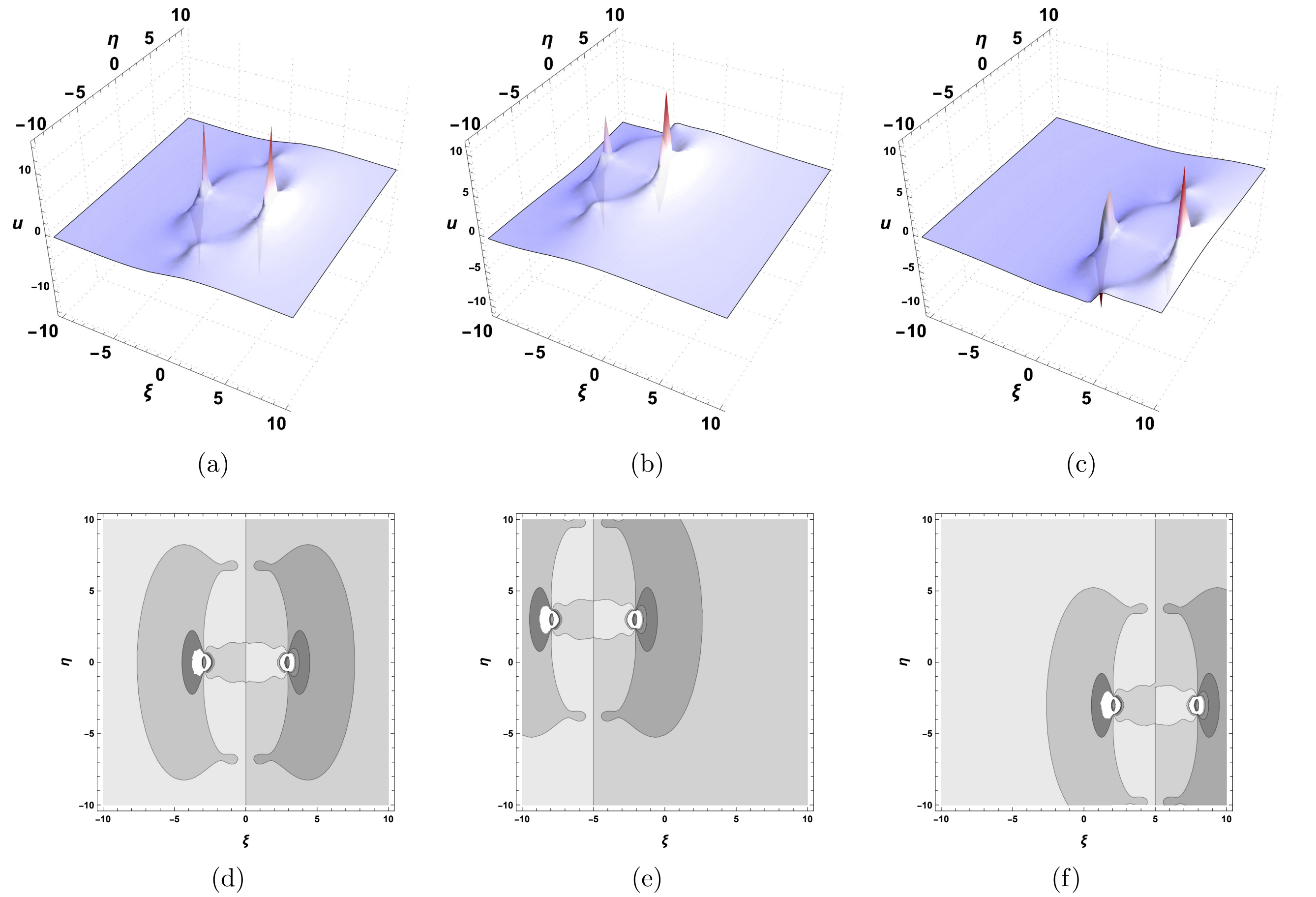

Figure 3: Third-order rogue wave for solution (29) with the function (28) having values: $\mu_1 = 4.5, \mu_2 = -3.5, \mu_3 = 10, \mu_4 = -3, \mu_5 = 10$, and center-controlled parameters as: **(a)** $\beta = 0, \gamma = 0$; **(b)** $\beta = -5, \gamma = 3$; and **(c)** $\beta = 5, \gamma = -3$. **(d-f)** are 2-D contours for **(a-c)** w.r.t $\xi\eta$-plane.

center-controlled parameters β and γ. Using direct computation, we constructed first, second, and third-order rogue waves with distinct values of center-controlled parameters and appropriate choices of different constants in the governed equation. Using the logarithmic transformation for the field u in the said equation, we obtained the bilinear equation in the auxiliary function f of the transformed variables ξ and η; after that, by applying the generalized formula for rogue waves formulated by the N-soliton using Hirota's bilinear approach, we got the rogue waves up to third order. We have depicted the dynamics for the rogue wave solutions with different center parameter values using symbolic system software *Mathematica*. We have shown that the singularities for the rogue waves occur concerning the center parameters β and γ. We have shown the dominating behavior of large rogue waves to the tiny rogue waves, which occur in many nonlinear phenomena. The equation studies wave motion in fluids and other weakly dispersive media and the propagation of long waves with small amplitudes in plasma physics. Rogue waves are also crucial in several nonlinear disciplines, including oceanography, fluid dynamics, dusty plasma, optical fibers, nonlinear dynamics, and other scientific subjects.

The presented work proposed a new (3+1)-dimensional generalized nonlinear equation, which generalizes the (3+1)-D Hirota bilinear equation and KP equation. Both are well-known equations that have applications in plasma physics, oceanography, and many other areas, so this generalized equation has scope for studying different wave solutions such as solitons, lumps, breathers, rogue waves, bright and dark solitons, and many others. Since we have applied the symbolic computational approach to obtain the rogue wave, there are many possibilities to study this equation with other techniques and methods discussed in the introduction section.

Acknowledgment

Conflict of interest

According to the authors, there is no conflict of interest.

Data availability statement

No data from other sources has been used in this study.

References

[1] Hereman W., Nuseir A.: Symbolic methods to construct exact solutions of nonlinear partial differential equations. Math Comput Simul, **43**, 13–27 (1997)

[2] Wazwaz, A.M.: Multiple soliton solutions for a (2+1)-dimensional integrable KdV6 equation. Commun. Nonlinear Sci. Numer. Simul. **15**, 1466–1472 (2010).

[3] Kumar, S., Mohan, B. , Kumar, R. : Lump, soliton, and interaction solutions to a generalized two-mode higher-order nonlinear evolution equation in plasma physics, Nonlinear Dyn, **110**, 693–704 (2022)

[4] Hirota R.: The direct method in soliton theory. Cambridge: Cambridge University Press, (2004).

[5] Wazwaz A.M.: The Hirota's direct method for multiple soliton solutions for three model equations of shallow water waves. Appl. Math. Comput. **201**, 489–503 (2008)

[6] Jiang Y., Tian B., Wang P. et al.: Bilinear form and soliton interactions for the modified Kadomtsev–Petviashvili equation in fluid dynamics and plasma physics. Nonlinear Dyn. **73**, 1343–1352 (2013)

[7] Kumar, S., Mohan, B.: A study of multi-soliton solu- tions, breather, lumps, and their interactions for Kadomtsev- Petviashvili equation with variable time coefficient using Hirota method. Phys. Scr. **96**(12), 125255 (2021)

[8] Kumar, S., Mohan, B.: A generalized nonlinear fifth-order KdV-type equation with multiple soliton solutions: Painlevé analysis and Hirota Bilinear technique, Phys. Scr. **97**, 125214, (2022)

[9] Huang Z.R., Tian B., Zhen H.L. et al.: Bäcklund transformations and soliton solutions for a (3+1)-dimensional B-type Kadomtsev–Petviashvili equation in fluid dynamics. Nonlinear Dyn. **80**, 1–7 (2015)

[10] Yan, X.W., Tian, S.F., Dong, M.J. et al.: Bäcklund transformation, rogue wave solutions and interaction phenomena for a (3+1)(3+1)-dimensional B-type Kadomtsev–Petviashvili–Boussinesq equation. Nonlinear Dyn. **92**, 709–720 (2018)

[11] Guan X., Liu W., Zhou, Q. et al.: Darboux transformation and analytic solutions for a generalized super-NLS-mKdV equation. Nonlinear Dyn. **98**, 1491–1500 (2019)

[12] Li H.M., Tian B., Xie X.Y.: Soliton and rogue-wave solutions for a (2 + 1)-dimensional fourth-order nonlinear Schrödinger equation in a Heisenberg ferromagnetic spin chain. Nonlinear Dyn. **86**, 369–380 (2016)

[13] Lan, Z.Z.: Rogue wave solutions for a higher-order nonlinear Schrödinger equation in an optical fiber, Applied Mathematics Letters **107**, 106382, (2020)

[14] Asaad, M.G., Ma, W.X.: Pfaffian solutions to a (3 + 1)-dimensional generalized B-type Kadomtsev–Petviashvili equation and its modified counterpart. Appl. Math. Comput. **218**, 5524–5542 (2012)

[15] Huang, Q.M., Gao, Y.T.: Wronskian, Pfaffian and periodic wave solutions for a (2+1)-dimensional extended shallow water wave equation. Nonlinear Dyn. **89**, 2855–2866 (2017)

[16] Nonlaopon K., Mann N., Kumar S., Rezaei S., Abdou M.A.: A variety of closed-form solutions, Painlevé analysis, and solitary wave profiles for modified KdV-Zakharov-Kuznetsov equation in (3+1)-dimensions, Results in Physics, 36, 105394 (2022)

[17] Kumar, S., Niwas, M.: New optical soliton solutions and a variety of dynamical wave profiles to the perturbed Chen–Lee–Liu equation in optical fibers. Opt Quant Electron **55**, 418 (2023)

[18] Kumar, S., Dhiman, S.K., Baleanu, D., Osman, M.S., Wazwaz, A.M.: Lie Symmetries, Closed-Form Solutions, and Various Dynamical Profiles of Solitons for the Variable Coefficient (2+1)-Dimensional KP Equations. Symmetry **14**, 597, (2022)

[19] Kumar, S., Rani S.: Invariance analysis, optimal system, closed-form solutions and dynamical wave structures of a (2+1)-dimensional dissipative long wave system, Phys. Scr. **96**, 125202, (2021)

[20] Kravchenko V.V.: Inverse Scattering Transform Method in Direct and Inverse Sturm-Liouville Problems. Frontiers in Mathematics, Birkhäuser, Cham., (2020)

[21] Zhou, X.: Inverse scattering transform for the time dependent Schrödinger equation with applications to the KPI equation, Commun. Math. Phys. **128**, 551–564 (1990)

[22] Zhang, R.F., Li, M.C., Fang, T., et al.: Multiple exact solutions for the dimensionally reduced p -gBKP equation via bilinear neural network method, Modern Physics Letters B, **36**(06), 2150590, (2022)

[23] Zhang, RF., Li, MC., Cherraf , A. et al. The interference wave and the bright and dark soliton for two integro-differential equation by using BNNM. Nonlinear Dyn, **111**, 8637–8646, (2023)

[24] Zhang, R.F., Bilige, S. : Bilinear neural network method to obtain the exact analytical solutions of nonlinear partial differential equations and its application to p-gBKP equation, Nonlinear Dyn, **95**, 3041–3048, (2019)

[25] Zhang, R.F., Li, M.C.: Bilinear residual network method for solving the exactly explicit solutions of nonlinear evolution equations. Nonlinear Dyn **108**, 521–531, (2022)

[26] Salah, M., Ragb O., Wazwaz A.M.: Efficient discrete singular convolution differential quadrature algorithm for solitary wave solutions for higher dimensions in shallow water waves, Waves in Random and Complex Media, (2022) DOI:10.1080/17455030.2022.2136420

[27] Nikolkina I., Didenkulova I.: Rogue waves in 2006–2010. Nat Hazards Earth Syst. Sci, **11**:2913–2924, (2011)

[28] Seadawy A.R., Rizvi S.T.R., Ahmed S, Bashir A.: Lump solutions, Kuznetsov–Ma breathers, rogue waves and interaction solutions for magneto electroelastic circular rod. Chaos Soli Fract, **163**:112563, (2022)

[29] Residori S., Onorato M., Bortolozzo U., Arecchi F.T.: Rogue waves: a unique approach to multidisciplinary physics, Contemporary Physics, **58**(1):53-69, (2017)

[30] Li L., Xie Y.: Rogue wave solutions of the generalized (3+1)-dimensional Kadomtsev–Petviashvili equation. Chaos Soli Fract, **147**:110935, (2021)

[31] Sun Y., Tian B., Liu L., Wu X.Y.: Rogue waves for a generalized nonlinear Schrödinger equation with distributed coefficients in a monomode optical fiber. Chaos Solit Fract, **107**:266-274, (2018)

[32] Cao, Y., Tian, H., Ghanbari, B.: On constructing of multiple rogue wave solutions to the (3+1)-dimensional Korteweg–de Vries Benjamin-Bona-Mahony equation, Phys. Scr. **96**, 035226, (2021)

[33] Zhang, R.F., Li, M.C., Gan, J.Y., et al.: Novel trial functions and rogue waves of generalized breaking soliton equation via bilinear neural network method,Chaos, Solitons & Fractals, **154**, 111692, (2022)

[34] Zhang, R.F., Li, M.C., Albishari, M., et al.: Generalized lump solutions, classical lump solutions and rogue waves of the (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada-like equation, Applied Mathematics and Computation, **403**, 126201, (2021)

[35] Yang, X., Zhang, Z., Wazwaz A.M., Wang, Z.: A direct method for generating rogue wave solutions to the (3+1)-dimensional Korteweg-de Vries Benjamin-Bona-Mahony equation, Physics Letters A **449**, 128355, (2022)

[36] Liu, W., Zhang, Y.: Multiple rogue wave solutions for a (3+1)-dimensional Hirota bilinear equation, Applied Mathematics Letters **98**, 184-190, (2019)

[37] Zhaqilao: Rogue waves and rational solutions of a (3+1)-dimensional nonlinear evolution equation, Physics Letters A **377**(42), 3021-3026, (2013)

[38] Guo, J., He, J., Li, M., Mihalache, D.: Multiple-order line rogue wave solutions of extended Kadomtsev–Petviashvili equation, Mathematics and Computers in Simulation **180**, 251-257, (2021)

[39] Zhang, H.Y., Zhang, Y.F.: Analysis on the M-rogue wave solutions of a generalized (3+1)-dimensional KP equation, Applied Mathematics Letters, **102**, 106145, (2020)

[40] Li, L., Xie, Y., Mei, L.: Multiple-order rogue waves for the generalized (2+1)-dimensional Kadomtsev–Petviashvili equation, Applied Mathematics Letters, **117**, 107079, (2021)

[41] Elboree, M.K.: Higher order rogue waves for the (3 + 1)-dimensional Jimbo–Miwa equation, International Journal of Nonlinear Sciences and Numerical Simulation, **23**, 7-8, (2022)

[42] Zhaqilao: A symbolic computation approach to constructing rogue waves with a controllable center in the nonlinear systems, Computers and Mathematics with Applications **75**(9), 3331-3342, (2018)

[43] Zhang, R.F., Li, M.C., Cherraf, A. et al. The interference wave and the bright and dark soliton for two integro-differential equation by using BNNM, Nonlinear Dyn, **111**, 8637–8646 (2023)

[44] Zhang R.F., Bilige, S., et al.: Bright-dark solitons and interaction phenomenon for p-gBKP equation by using bilinear neural network method, Phys. Scr. **96**, 025224, (2021)

[45] Zhang, R., Bilige, S. , Chaolu, T.: Fractal Solitons, Arbitrary Function Solutions, Exact Periodic Wave and Breathers for a Nonlinear Partial Differential Equation by Using Bilinear Neural Network Method, J Syst Sci Complex,**34**, 122–139, (2021)

[46] Wazwaz, A.M., Albalawi, W., El-Tantawy, S.A.: Optical envelope soliton solutions for coupled nonlinear Schrödinger equations applicable to high birefringence fibers, Optik, **255**, 168673, (2022)

[47] Kumar, S., Kumar A., Mohan, B.: Evolutionary dynamics of solitary wave profiles and abundant analytical solutions to a (3+1)-dimensional burgers system in ocean physics and hydrodynamics, Journal of Ocean Engineering and Science,**8**(1), 1-14, (2023)

[48] Zhang, R.F, Li, M.C., Al-Mosharea, E. et al.: Rogue waves, classical lump solutions and generalized lump solutions for Sawada–Kotera-like equation, International Journal of Modern Physics B, **36**(05), 2250044, (2022)

[49] Wazwaz, A.M., Hammad, M.A., El-Tantawy,S.A. : Bright and dark optical solitons for (3 + 1)-dimensional hyperbolic nonlinear Schrödinger equation using a variety of distinct schemes, Optik, **270**, 170043, (2022)

[50] Kaur, L., Wazwaz, A.M.: Optical soliton solutions of variable coefficient Biswas–Milovic (BM) model comprising Kerr law and damping effect,Optik, **266**,169617, (2022)

[51] Wazwaz, A.M. : New (3+1)-dimensional Painlevé integrable fifth-order equation with third-order temporal dispersion, Nonlinear Dyn, **106**, 891–897 (2021)

[52] Wazwaz, A.M. : Painlevé integrability and lump solutions for two extended (3 + 1)- and (2 + 1)-dimensional Kadomtsev–Petviashvili equations, Nonlinear Dyn, **111**, 3623–3632 (2023)

[53] Baldwin D., Hereman W.: Symbolic software for the Painlevé test of nonlinear differential ordinary and partial equations, Journal of Nonlinear Mathematical Physics, **13**(1), 90-110 (2006)

[54] Weiss J, Tabor M, Carnevale G. The Painlevé property for partial differential equations. J. Math. Phys. **24**, 522–526 (1983)