Newly formed center-controlled rouge wave and lump solutions of a generalized (3+1)-dimensional KdV-BBM equation via symbolic computation approach

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Abstract: In this article, we investigate the generalized (3+1)-dimensional KdV-Benjamin-Bona-Mahony equation governed with constant coefficients. It applies the Painlevé analysis to test the complete integrability of the concerned KdV-BBM equation. The symbolic computational approach provides first-order, second-order rogue wave and lump solutions with center-controlled parameters. The rogue waves localized in space and time have a significant amplitude, and lumps are of rational form solution, localized decaying solutions in all space directions rationally. Utilizing a symbolic computation approach, we get the bilinear equation of the KdV-Benjamin-Bona-Mahony equation and show the center-controlled rogue waves and lumps. We employ the symbolic system software *Mathematica* to do the symbolic computations, form the first and second-order rogue waves, and lump solutions with appropriate values of constant coefficients. The KdV-Benjamin-Bona-Mahony equation analyses the evolution of long waves with modest amplitudes propagating in plasma physics and the motion of waves in fluids and other weakly dispersive mediums. Moreover, rogue waves and lumps occur in several scientific areas, such as fluid dynamics, optical fibers, dusty plasma, oceanography, water engineering, and other nonlinear sciences.

Keywords: Bilinearization; Cole-Hopf transformation; Painlevé analysis; Generalized KdV-BBM equation; Center-controlled solutions.

1 Introduction

A nonlinear partial differential equation (PDE) represents a wide variety of physical systems, including fluid dynamics, plasma physics, shallow water waves, and oceanography. It is a mathematical and physical term for an equation with partial derivatives and nonlinear components. PDEs have been used to solve various conjectures, including the Poincaré conjecture and the Calabi conjecture. Nonlinear PDEs cannot be solved using a generic method; hence each equation is examined separately. Pursuing novel, precise solutions to nonlinear PDEs, prevalent in several nonlinear scientific fields, is always a fascinating subject. Several approaches have been put forth to deal with nonlinear PDEs, including the Hirota's bilinearization method [1–5], the Darboux transformation [6–8], simplified Hirota's technique [9–11], the Bäcklund transformation [12,13], the Lie symmetry analysis [14–19], the Pfaffian technique [20,21], Inverse scattering method [22,23], and other techniques.

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Rogue waves [24–30], sometimes known as enormous solitary waves that "come from nowhere" in the water, are localized in space-time with significant amplitude. They can severely harm people since they are unexpected and extensive. As a result, exploring the developing mechanism of rogue waves is crucial, which has garnered substantial interest from several scholars. In 2018, Zhaqilao [31] gave a direct Symbolic Computational Approach (SCA) technique to generate higher-order rogue waves in nonlinear systems with center-controlled parameters. Moreover, in 2021, Y. Cao, H. Tian, and B. Ghanbari [32] constructed higher-order rogue wave solutions of (3+1)-D KdV-Benjamin-Bona-Mahony (KdV-BBM) equation utilizing SCA. Furthermore, in 2022, X. Yang, Z. Zhang, et al. [33] gave a direct method for constructing higher-order rogue waves of the (3+1)-D KdV-BBM equation from N-soliton of the Hirota method using long wave limit technique. Several mathematicians and researchers have shown curiosity in lump waves [35–41] as the appropriate prototypes to sport rogue waves in an optical medium, fluid dynamics, and plasmas. Lump wave solutions are localized in nature, rationally decaying in all space directions. Manakov et al. [42] found a lump wave and found that the lump wave interactions do not practice phase shifts. Subsequently, Satsuma and Ablowitz [43] described the interactions of lumps from N-solitons of KP and the (2+1)-D Schrödinger equation. After that, in literature, numerous high-dimensional nonlinear PDEs also admit lumps, such as the three-wave resonant interaction equation [44], shallow water-like equation [45], two-mode generalized evolution equation [46], extended KP equation [47], and many other equations.
A Symbolic Computational Approach [48–55] is a valuable tool for creating rogue wave solutions of an integrable nonlinear PDE among the several approaches to studying nonlinear PDEs. SCA offers a straightforward method for locating higher-order rogue waves. To produce the rogue waves of the nonlinear PDEs, Zhaqilao's SCA [31] includes a phase in which the equation is first transformed into a bilinear form, as described by Hirota, and then uses a dependent variable transformation. We may streamline this procedure and compute the various nonlinear PDE solutions by skipping the bilinear form stage and immediately converting the equation into an elucidated form.
Verifying a PDE's integrability is crucial since the concerned approach is applied to integrable nonlinear PDE. While integrable PDEs have exponentially localized solutions, studying their integrability to nonlinear PDEs helps produce solitons, lumps, rogue waves, and other solutions. We assess the entire integrability of a nonlinear PDE using the Painlev'e test. We analyze this using symbolic system software *Mathematica*. Baldwin and Hereman [10] constructed an algorithm and provided a symbolic computation for Painlevé analysis using the WTC-Krushkal method [56].

In this article, we investigate the generalized (3+1)-dimensional nonlinear KdV-BBM [32–34] equation

$$u_{xt} + \alpha_1(uu_x)_x + \alpha_2 u_{xxxx} - \alpha_3 u_{xxxt} + \alpha_4 u_{yy} - \alpha_5 u_{zz} = 0, \tag{1}$$

where the coefficients $\alpha_i; 1 \leq i \leq 5$ are real parameters. The equation (1) generalizes the well-known equations as

- When $\alpha_3 = \alpha_4 = \alpha_5 = 0$, equation (1) gives to the KdV equation as

$$u_t + \alpha_1 uu_x + \alpha_2 u_{xxx} = 0 \tag{2}$$

- When $\alpha_3 = \alpha_5 = 0$, equation (1) becomes the (2+1)-D KP equation as

$$u_{xt} + \alpha_1(uu_x)_x + \alpha_2 u_{xxxx} + \alpha_4 u_{yy} = 0 \tag{3}$$

- When $\alpha_3 = 0$, equation (1) gives (3+1)-D KP equation as

$$u_{xt} + \alpha_1(uu_x)_x + \alpha_2 u_{xxxx} + \alpha_4 u_{yy} - \alpha_5 u_{zz} = 0 \tag{4}$$

It is essential to address the issue of describing physically convincing answers to the equation (1). This equation analyzes the transition of long waves with modest amplitudes propagating in plasma physics. It determines whether the transverse dynamics should be considered and can be used to analyze the motion of waves in fluids and other weakly dispersive mediums. By simplifying the symbolic computational approach, we drive the first-order, second-order rogue wave and lump solutions. Due to the ignorance of nonlinearity and dispersion, solitons are formed; thus, solitons, lumps, and rogue waves play a crucial role in studying shallow water waves. By applying the Painlevé analysis, we get complete integrability of the KdV-BBM equation, which confirms the existence of such solutions as rogue waves, lumps, solitons, and others. We bilinearize of the said equation and compute the center-controlled rogue wave solutions with center parameters for first-order and second-order and the center-controlled lump solutions with two sets of coefficient values. Furthermore, the dynamical study of the obtained solution shows the existence of rogue waves and lumps in fluid dynamics, optical fibers, dusty plasma, oceanography, water engineering, and other nonlinear sciences.
The article is organized as follows: Coming section investigates the integrability of the KdV-BBM equation using Painlevé analysis. In Section 3, we simplify the symbolic computational approach and apply it to the KdV-BBM equation to form an bilinear equation of the said equation. Section 4 computes the center-controlled first-order and second-order rogue wave solutions and shows the obtained solution's dynamics with different center parameter values. In section 5, we obtain the center-controlled lump solutions with two sets of coefficient values and depict the dynamics for the same with different center parameter values. Section 6 discusses the results of obtained solutions and their graphics. The last section concludes the work and investigation.

2 Painlevé test analysis

In Painlevé analysis, we consider the solution of the Eq. (1) as a Laurent's series about the manifold $\rho(x, y, z, t)$, to investigate the integrability as

$$u(x, y, z, t) = \sum_{j=0}^{\infty} u_j(x, y, z, t)\rho^{j-\lambda}, \tag{5}$$

where λ is a positive number, and $u_j = u_j(x, y, z, t); j = 0, 1, 2, ...;$. On substituting the Eq. (5) into Eq. (1), we compute λ by resembling the dominance terms as

$$\lambda = 2.$$

Also, we encounter the leading order behavior concerning the resonance j as

$$u_0 = \frac{12\rho_x(\alpha_3\rho_t - \alpha_2\rho_x)}{\alpha_1}; \quad j = -1, 4, 5, 6.$$

Due to the irrational choice, the resonance $j = -1$ for singular manifold $\rho(x, y, z, t) = 0$ occurs. The expressions for $u_j; j = 1, 2, ...,$ exist explicitly with random choices for u_4, u_5 and u_6. Resonances j satisfy the condition for compatibility identically. Furthermore, it exhibits that the concerned Eq. (1) is entirely Painlevé integrable.

3 Simplification of Symbolic Computation Approach

Let us consider a (3+1)-D nonlinear PDE as

$$P(u, u_t, u_{xt}, u_{yt}, u_{zt}, u_x, u_{xx}, u_{xy}, u_{xz}, u_z, u_{zx}, \cdots) = 0, \tag{6}$$

where subscripts are the partial derivatives w.r.t. the variables x, y, z, and t.
Step 1: We use a Painlevé transformation

$$u = T(F(\xi, z)), \tag{7}$$

where $\xi = \xi(x, y, t)$ and F is a auxiliary function of dependent variables.
Step 2: By substituting the transformation (7) into the Eq. (6), we get an bilinear equation in $F(\xi, z)$

$$Q(F(\xi, z)) = 0 \tag{8}$$

Step 3:
(i) For center-controlled rogue waves, we assume the auxiliary function F as

$$F(\xi, z) = \widehat{f}(\xi, z, \beta, \gamma) = f_{k+1} + 2\beta z p_k + 2\gamma \xi q_k + (\beta^2 + \gamma^2) f_{k-1}, \tag{9}$$

with

$$\begin{aligned}
f_k(\xi, z) &= \sum_{n=0}^{k(k+1)/2} \sum_{j=0}^{n} a_{(k(k+1)-2n),2j} z^{2j} \xi^{(k(k+1)-2n)}, \\
p_k(\xi, z) &= \sum_{n=0}^{k(k+1)/2} \sum_{j=0}^{n} b_{(k(k+1)-2n),2j} z^{2j} \xi^{(k(k+1)-2n)}, \\
q_k(\xi, z) &= \sum_{n=0}^{k(k+1)/2} \sum_{j=0}^{n} c_{(k(k+1)-2n),2j} z^{2j} \xi^{(k(k+1)-2n)},
\end{aligned}$$

$f_0 = 1, f_{-1} = p_0 = q_0 = 0$, where $a_{l,m}, b_{l,m}, c_{l,m}$ with $l, m \in \{0, 2, \cdots, k(k+1)\}$ and β, γ are the real parameters. The values for $a_{l,m}, b_{l,m}, c_{l,m}$ can be determined, and the constants β, γ control the center of the wave.
(ii) For generating the center-controlled lump solution, we consider the auxiliary function F as a rational solution

$$\begin{aligned}
F(\xi, z) &= G^2 + H^2 + a_7, \\
G(\xi, z) &= a_1(\xi - \gamma) + a_2(z - \beta) + a_3, \\
H(\xi, z) &= a_4(\xi - \gamma) + a_5(z - \beta) + a_6,
\end{aligned} \tag{10}$$

where $a_i; 1 \le i \le 7$ are the constants and β, γ are the center-controlled parameters.

3.1 Application of SCA to KdV-BBM equation

Considering $\xi(x, y, t) = x + gy - ht$ in equation (1), we get

$$\alpha_1(u_\xi^2 + u u_{\xi\xi}) + \alpha_2 u_{\xi\xi\xi\xi} + h\alpha_3 u_{\xi\xi\xi\xi} + g^2 \alpha_4 u_{\xi\xi} - h u_{\xi\xi} - \alpha_5 u_{zz} = 0, \tag{11}$$

where g and h are the constants. Now, considering the phase variable ϕ_i in the equation (11) as

$$\phi_i = p_i \xi - w_i z, \tag{12}$$

where p_i and w_i for $i = 1, 2, 3, ...,$ are the real parameters and dispersion coefficients respectively. On substituting $u(\xi, z) = e^{\phi_i}$ in the linear terms of Eq. (11), we compute the dispersion as

$$w_i = \pm \frac{p_i \sqrt{\alpha_4 g^2 + \alpha_3 h p_i^2 + \alpha_2 p_i^2 - h}}{\sqrt{\alpha_5}}. \tag{13}$$

Assuming the Cole-Hopf transformation

$$u(\xi,z)=R(logF)_{\xi\xi} \tag{14}$$

with R as a non-zero parameter and $F=F(\xi,z)$ as a dependent variable function. We substitute the transformation (14) and equation (13) with $F(\xi,z)=1+e^{p_1\xi-w_1z}$ in equation (11), and solve for R, we get

$$R=\frac{12\,(\alpha_2+\alpha_3h)}{\alpha_1}$$

To compute the different solutions for the equation (11), we consider the dependent variable transformation as

$$u(\xi,z)=u_0+\frac{12(\alpha_2+\alpha_3h)}{\alpha_1}(logF)_{\xi\xi}. \tag{15}$$

Substitution of transformation (15) into equation (11) converts to the bilinear equation in $F(\xi,z)$ as

$$\begin{aligned}hF_\xi^2-hFF_{\xi\xi}-\alpha_1u_0F_\xi^2+\alpha_1u_0FF_{\xi\xi}+3\alpha_2F_{\xi\xi}^2-4\alpha_2F_\xi F_{\xi\xi\xi}+\alpha_2FF_{\xi\xi\xi\xi}-4\alpha_3hF_\xi F_{\xi\xi\xi}\\+3\alpha_3hF_{\xi\xi}^2+\alpha_3hFF_{\xi\xi\xi\xi}-\alpha_4g^2F_\xi^2+\alpha_4g^2FF_{\xi\xi}+\alpha_5F_{\mathrm{z}}^2-\alpha_5FF_{\mathrm{zz}}=0.\end{aligned} \tag{16}$$

4 Center-controlled rogue wave solutions

4.1 First-order rogue wave solution

To construct a first-order rogue wave, we choose dependent variable function $F(\xi,z)$ for $k=0$ in equation (9) as

$$F(\xi,z)=a_{2,0}\xi^2+a_{0,2}z^2+a_{0,0}. \tag{17}$$

On substituting Eq. (17) into the bilinear Eq. (16), and equating all the coefficients of different powers of $\xi^rz^s;r,s\in Z$ to zero, we get a system of equations

$$-2\alpha_5a_{0,2}a_{2,0}-2\alpha_4g^2a_{2,0}^2+2ha_{2,0}^2-2\alpha_1u_0a_{2,0}^2 = 0 \tag{18}$$

$$2\alpha_5a_{0,2}^2+2\alpha_4g^2a_{0,2}a_{2,0}-2ha_{0,2}a_{2,0}+2\alpha_1u_0a_{0,2}a_{2,0} = 0 \tag{19}$$

$$12\alpha_2a_{2,0}^2-2\alpha_5a_{0,0}a_{0,2}+2\alpha_4g^2a_{0,0}a_{2,0}+12\alpha_3ha_{2,0}^2-2ha_{0,0}a_{2,0}+2\alpha_1u_0a_{0,0}a_{2,0} = 0 \tag{20}$$

On solving this system , we obtain the constants as

$$\begin{aligned}a_{0,0} &= \frac{3(\alpha_2+\alpha_3h)a_{2,0}}{-\alpha_4g^2+h-\alpha_1u_0},\\ a_{0,2} &= \frac{(-\alpha_4g^2+h-\alpha_1u_0)a_{2,0}}{\alpha_5},\\ a_{2,0} &= a_{2,0}.\end{aligned} \tag{21}$$

Thus, the equation (17) with values in (21) becomes

$$F(\xi,z)=\widehat{f}_1(\xi,z,\beta,\gamma)=\left((\xi-\beta)^2+\frac{3\,(\alpha_2+\alpha_3h)}{-\alpha_4g^2+h-\alpha_1u_0}+\frac{(z-\gamma)^2\left(-\alpha_4g^2+h-\alpha_1u_0\right)}{\alpha_5}\right)a_{2,0}, \tag{22}$$

which is a solution of equation (16) with center controlled parameters β and γ. By substituting equation (22) into (15), we get a rogue wave solution as

$$u(\xi,z)=u_0+\frac{24\,(\alpha_2+\alpha_3h)\left(-(\beta-\xi)^2-\frac{3(\alpha_2+\alpha_3h)}{\alpha_4g^2-h+\alpha_1u_0}+\frac{(z-\gamma)^2\left(-\alpha_4g^2+h-\alpha_1u_0\right)}{\alpha_5}\right)}{\alpha_1\left((\beta-\xi)^2-\frac{3(\alpha_2+\alpha_3h)}{\alpha_4g^2-h+\alpha_1u_0}+\frac{(z-\gamma)^2(-\alpha_4g^2+h-\alpha_1u_0)}{\alpha_5}\right)^2} \tag{23}$$

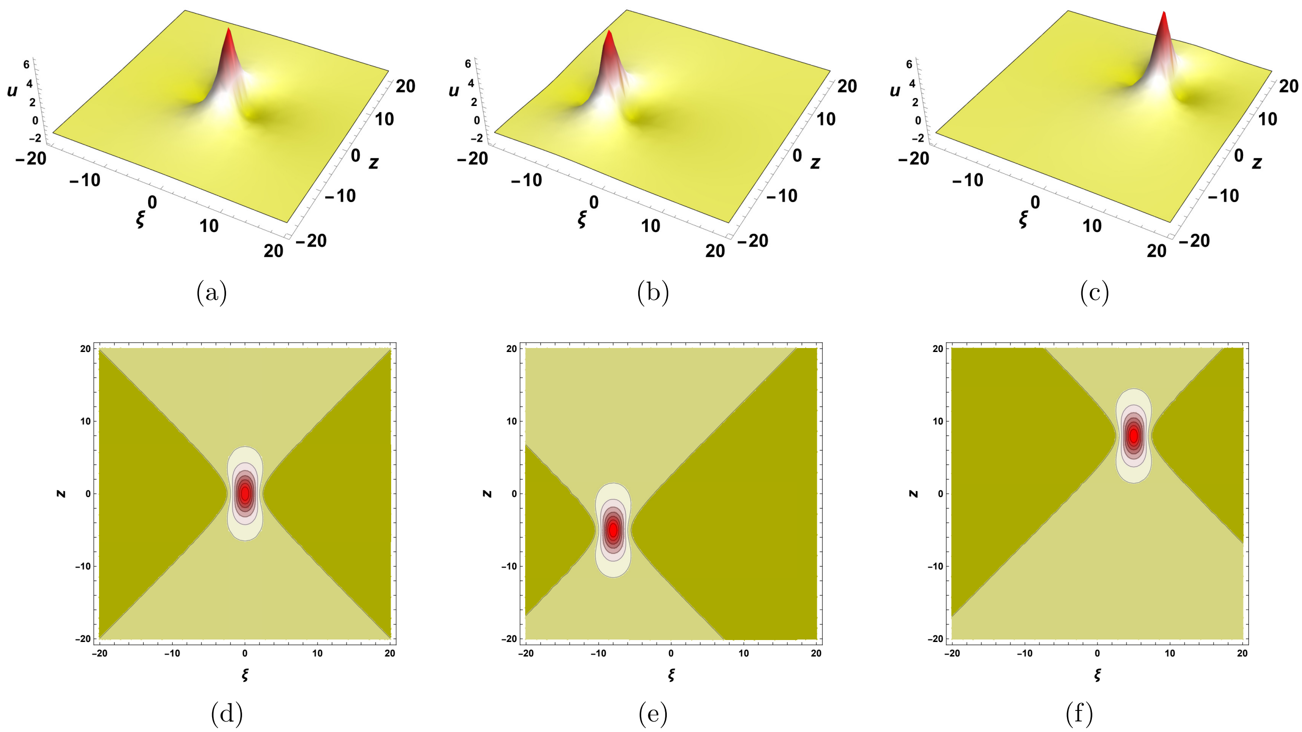


Figure 1: First-order rogue wave for solution (23) with the function (22) having values: $u_0 = -1, \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 1, g = h = 1$, and center-controlled parameters as: **(a)** $\beta = 0, \gamma = 0$; **(b)** $\beta = -8, \gamma = -5$; and **(c)** $\beta = 5, \gamma = 8$. **(d-f)** are 2-D contours for **(a-c)** w.r.t ξz-plane.

4.2 Second-order rogue wave solution

For a second-order rogue wave, we consider dependent variable function $F(\xi, z)$ for $k = 1$ in equation (9) as

$$F(\xi, z) = \xi^6 a_{6,0} + \xi^4 a_{4,0} + \xi^2 a_{2,0} + z^6 a_{0,6} + \xi^2 z^4 a_{2,4} + z^4 a_{0,4} + \xi^4 z^2 a_{4,2} + \xi^2 z^2 a_{2,2} + z^2 a_{0,2} + a_{0,0}. \quad (24)$$

Substituting Eq. (24) into the bilinear Eq. (16), and equating all the coefficients of different powers of $\xi^r z^s; r, s \in Z$ to zero, gives a system of equations. Solving this system of equations results the constants as

$$a_{0,0} = \frac{625\alpha_5 (\alpha_2 + \alpha_3 h)^3 a_{4,2}}{(\alpha_4 g^2 - h + \alpha_1 u_0)^4}, \quad a_{0,2} = \frac{475 (\alpha_2 + \alpha_3 h)^2 a_{4,2}}{3 (\alpha_4 g^2 - h + \alpha_1 u_0)^2}, \quad a_{0,4} = \frac{17 (\alpha_2 + \alpha_3 h) a_{4,2}}{3\alpha_5}$$

$$a_{0,6} = \frac{(\alpha_4 g^2 - h + \alpha_1 u_0)^2 a_{4,2}}{3\alpha_5^2}, \quad a_{2,0} = \frac{125\alpha_5 (\alpha_2 + \alpha_3 h)^2 a_{4,2}}{3 (\alpha_4 g^2 - h + \alpha_1 u_0)^3}, \quad a_{2,2} = \frac{-30 (\alpha_2 + \alpha_3 h) a_{4,2}}{(\alpha_4 g^2 - h + \alpha_1 u_0)}$$

$$a_{2,4} = \frac{-(\alpha_4 g^2 - h + \alpha_1 u_0) a_{4,2}}{\alpha_5}, \quad a_{4,0} = \frac{25\alpha_5 (\alpha_2 + \alpha_3 h) a_{4,2}}{3 (\alpha_4 g^2 - h + \alpha_1 u_0)^2}$$

$$a_{6,0} = \frac{-\alpha_5 a_{4,2}}{3(\alpha_4 g^2 - h + \alpha_1 u_0)} \quad (25)$$

where $a_{4,2}$ is an arbitrary constant. Thus, the equation (17) with values in (25) becomes

$$F(\xi, z) = \widehat{f}_2(\xi, z, \beta, \gamma) = \frac{a_{4,2}}{3}\Big(\frac{25\alpha_5(\beta-\xi)^4 A}{B^2} - \frac{\alpha_5(\beta-\xi)^6}{B} + \frac{125\alpha_5(\beta-\xi)^2 A^2}{B^3} + \frac{1875\alpha_5 A^3}{B^4} - \frac{3(\beta-\xi)^2(z-\gamma)^4 B}{\alpha_5} - \frac{90(\beta-\xi)^2 A(z-\gamma)^2}{B} + \frac{(z-\gamma)^6 B^2}{\alpha_5^2} + \frac{475A^2(z-\gamma)^2}{B^2} + \frac{17A(z-\gamma)^4}{\alpha_5} + 3(\beta-\xi)^4(z-\gamma)^2\Big), \quad (26)$$

where $A = (\alpha_2 + \alpha_3 h)$ and $B = (\alpha_4 g^2 - h + \alpha_1 u_0)$, which is a solution of equation (16) with center controlled parameters β and γ. By substituting equation (26) into (15), we get a second-order rogue wave solution as

$$u(\xi, z) = u_0 + \frac{12(\alpha_2 + \alpha_3 h)}{\alpha_1}(log\widehat{f}_2(\xi, z, \beta, \gamma))_{\xi\xi}. \quad (27)$$

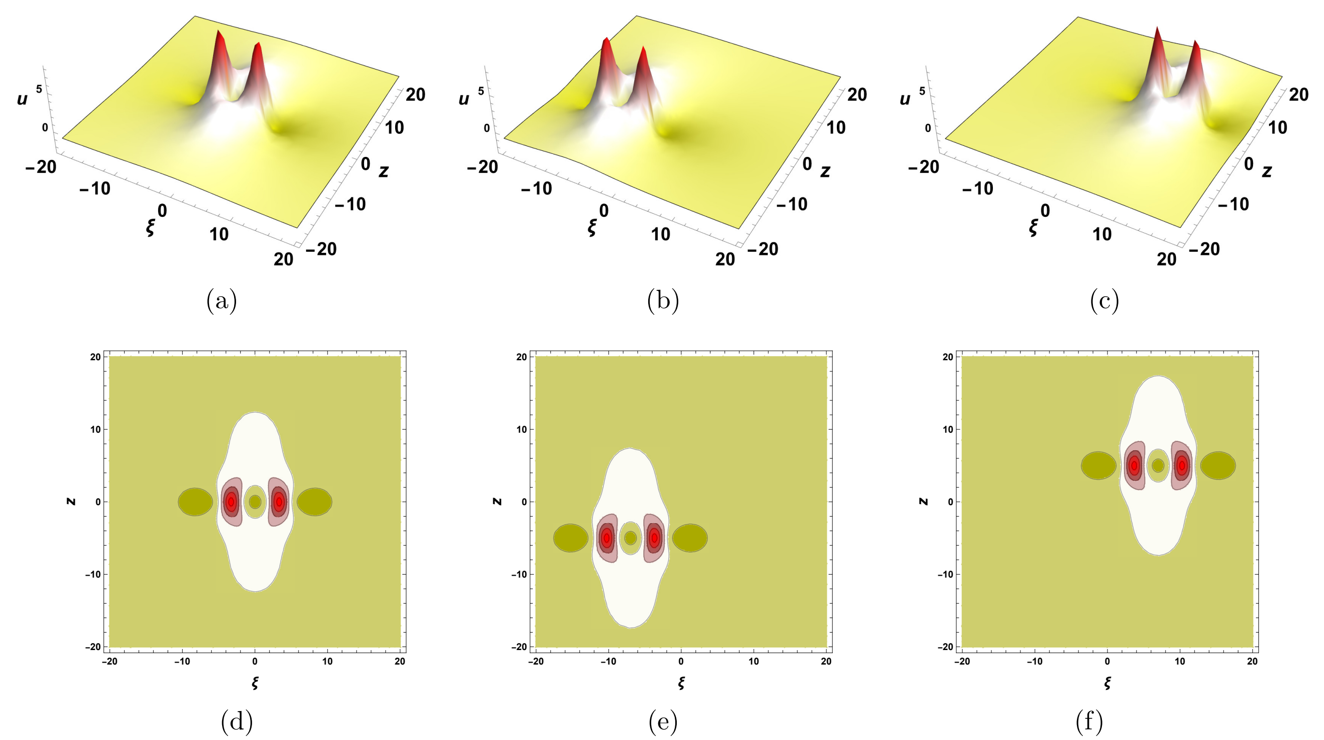


Figure 2: Second-order rogue wave for solution (27) with the function (22) having values: $u_0 = -1, \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 1, g = h = 1$, and center-controlled parameters as: **(a)** $\beta = 0, \gamma = 0$; **(b)** $\beta = -7, \gamma = -5$; and **(c)** $\beta = 7, \gamma = 5$. **(d-f)** are 2-D contours for **(a-c)** w.r.t ξz-plane.

5 Center-controlled lump solutions

To construct center-controlled lump, we consider the function $F(\xi, z)$ as Eq. (10)

$$\begin{aligned} F(\xi, z) &= G^2 + H^2 + a_7, \\ G(\xi, z) &= a_1(\xi - \gamma) + a_2(z - \beta) + a_3, \\ H(\xi, z) &= a_4(\xi - \gamma) + a_5(z - \beta) + a_6, \end{aligned} \tag{28}$$

where the constants $a_i; 1 \le i \le 7$ need to be determined. By substituting the Eq. (28) into the bilinear Eq. (16), and equating all the coefficients of terms with powers of $\xi^r z^s; r, s \in Z$ to zero. Furthermore, we can get different solutions by solving the obtained system of equations with appropriate choices. We are showing two sets as

1. $a_3 = a_6 = a_7 = 0, a_5 = \frac{-a_1 a_2}{a_4}, a_4 = a_4 \neq 0, a_1 = a_1, a_2 = a_2,$
2. $a_1 = a_5 = a_6 = 0, a_4 = \pm \frac{a_2 \sqrt{\alpha_5}}{\sqrt{h - u_0 \alpha_1 - g^2 \alpha_4}}, a_2 = a_2, a_3 = a_3, a_7 = a_7,$

Therefore, we compute a center-controlled lump solution by putting Eq. (28) with a solution set into Eq. (15) as

$$u(\xi, z) = u_0 - \frac{24 a_4^2 (\alpha_2 + \alpha_3 h) \left(a_4^2 (\gamma - \xi)^2 - a_2^2 (z - \beta)^2\right)}{\alpha_1 \left(a_4^2 (\gamma - \xi)^2 + a_2^2 (z - \beta)^2\right)^2} \tag{29}$$

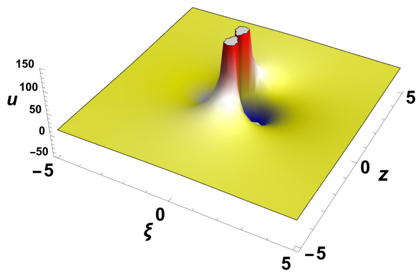
$$u(\xi, z) = u_0 + \frac{24 a_2^2 \alpha_5 (\alpha_2 + \alpha_3 h) \left(\alpha_5 a_2^2 \left(-(\gamma - \xi)^2\right) + a_7 T + (a_2(z - \beta) + a_3)^2 T\right)}{\alpha_1 T^2 \left(\frac{\alpha_5 a_2^2 (\gamma - \xi)^2}{T} + (a_2(z - \beta) + a_3)^2 + a_7\right)^2} \tag{30}$$

where $T = \left(-\alpha_4 g^2 + h - \alpha_1 u_0\right)$, for both solution sets as above, respectively.

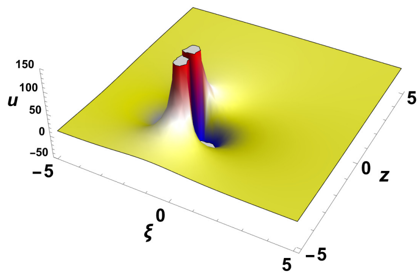
6 Results and discussion

Due to the complete integrability of the KdV-Benjamin-Bona-Mahony equation, it can possess different solutions, such as solitons, rogue waves, lumps, and others. We obtained the center-controlled first-order, second-order rogue waves and center-controlled lumps for the KdV-BBM equation with appropriate choices of parameters and displayed the dynamics for the computed solutions. Furthermore, the discussion of the results is as

- Figure 1 depicts the center-controlled first-order rogue wave solution due to the singularity $\xi = \gamma$ along with the parameters $u_0 = -10, \alpha_3 = -1, \alpha_1 = \alpha_2 = \alpha_4 = \alpha_5 = 1, g = h = 1$, and center parameters as $\beta = 0, \gamma = 0$; $\beta = -8, \gamma = 0$; and $\beta = 5, \gamma = 8$, for (a), (b), and (c), respectively.
- In figure 2, we illustrate the center-controlled second-order rogue wave solution due to the singularity around ξ that moves depending on the value of $\xi = \gamma$, along with the parameters $u_0 = -1, \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 1, g = h = 1$, and center parameters as $\beta = 0, \gamma = 0$; $\beta = -7, \gamma = -5$; and $\beta = 7, \gamma = 5$, for (a), (b), and (c), respectively.
- Figure 3 shows the center-controlled lump solution due to the singularity $\xi = \gamma$ along with the parameters $u_0 = 10, a_2 = a_4 = 0.5, \alpha_1 = \alpha_3 = 1, \alpha_2 = 2$ and center parameters $\beta = 0, \gamma = 0$; $\beta = -3, \gamma = -1$; and $\beta = 3, \gamma = 2$ for (a), (b), and (c), respectively.
- In figure 4, we depict the center-controlled lump solution due to the singularity $\xi = \gamma$ along with the parameters $u_0 = 5, a_2 = 0.9, a_3 = a_7 = 0.5, \alpha_1 = \alpha_2 = 1, \alpha_3 = \alpha_4 = \alpha_5 = 1, g = h = 1$, and center parameters $\beta = 0, \gamma = 0$; $\beta = -8, \gamma = -5$; and $\beta = 3, \gamma = 8$ for (a), (b), and (c), respectively.



(a)



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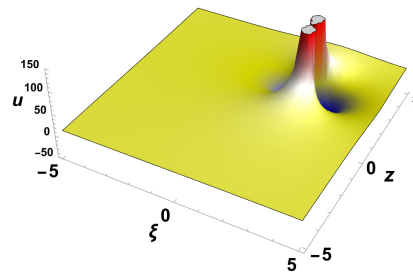
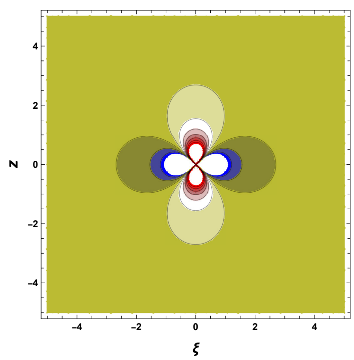
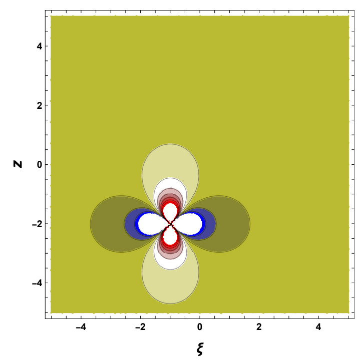


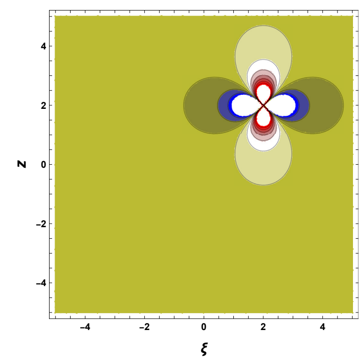
(c)



(d)



(e)



(f)

Figure 3: Lump for solution (29) with the function (28) having values: $u_0 = 10, a_2 = a_4 = 0.5, \alpha_1 = \alpha_3 = 1, \alpha_2 = 2, h = 1$, and center-controlled parameters as: **(a)** $\beta = 0, \gamma = 0$; **(b)** $\beta = -2, \gamma = -1$; and **(c)** $\beta = 2, \gamma = 2$. **(d-f)** are 2-D contours for **(a-c)** w.r.t ξz-plane.

7 Conclusions

This article investigated a generalized (3+1)-dimensional KdV-Benjamin-Bona-Mahony equation with constant coefficients. We used the simplified symbolic computational approach to obtain center-controlled first-order, second-order rogue wave, and center-controlled lump solutions. Painlevé analysis is used to check the complete integrability of the investigated KdV-BBM equation. We examined the rogue waves and lumps for different values of center-controlled parameters and appropriate choices of different present parameters in the KdV-BBM equation. Using the symbolic computation approach, we obtained the bilinear equation in the auxiliary function of the KdV-BBM equation, then obtained the first-order, second-order rogue wave and lump solutions, and depicted the dynamics for the solutions with different center parameter values using symbolic system software *Mathematica*. We have shown that the singularities for the rogue and lump waves occur concerning the center parameters β and γ. The KdV-BBM equation analyses the development of long waves with modest amplitudes propagating in plasma physics and the motion of waves in fluids and other weakly dispersive mediums. Moreover, rogue wave and lump solutions are essential in several nonlinear fields, such as fluid dynamics, optical fibers, dusty plasma, oceanography, water engineering, and other scientific areas.

Conflict of interest

There is no conflict of interest, according to the authors.

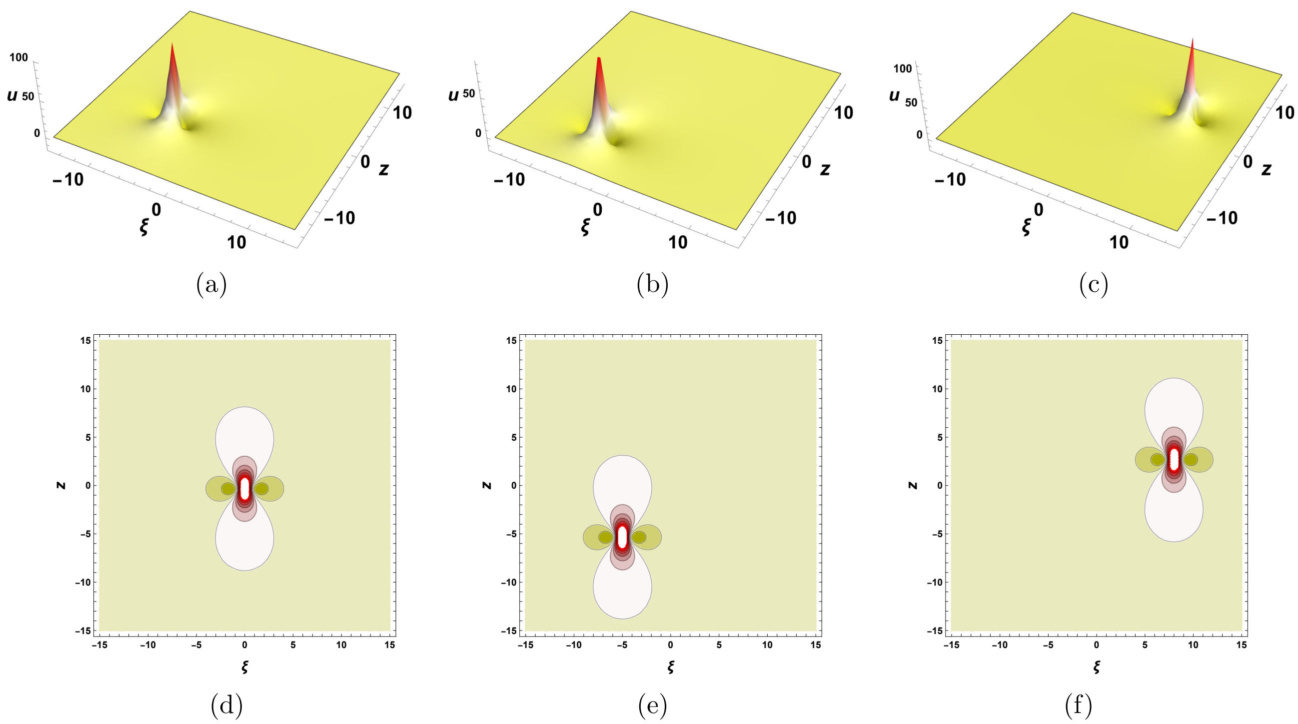


Figure 4: Lump for solution (30) with the function (28) having values: $u_0 = -1, a_2 = 0.9, a_3 = 0.3, a_7 = 0.5, \alpha_1 = \alpha_2 = \alpha_4 = \alpha_5 = 1, \alpha_3 = 2, g = h = 1$, and center-controlled parameters as: **(a)** $\beta = 0, \gamma = 0$; **(b)** $\beta = -5, \gamma = -5$; and **(c)** $\beta = 3, \gamma = 8$. **(d-f)** are 2-D contours for **(a-c)** w.r.t ξz-plane.

Data availability statement

No new data were created or analysed in this study.

References

[1] Hirota R. The direct method in soliton theory. Cambridge: Cambridge University Press, 2004.

[2] Wazwaz AM. The Hirota's direct method for multiple soliton solutions for three model equations of shallow water waves. Appl Math Comput. 2008; 201:489-503.

[3] Jiang Y, Tian B, Wang P et al. Bilinear form and soliton interactions for the modified Kadomtsev–Petviashvili equation in fluid dynamics and plasma physics. Nonlinear Dyn. 2013; 73:1343–1352.

[4] Kumar S, Mohan B. A study of multi-soliton solutions, breather, lumps, and their interactions for Kadomtsev-Petviashvili equation with variable time coefficient using Hirota method. Phys Scr. 2021; 96(12):125255.

[5] Kumar S, Mohan B. A generalized nonlinear fifth-order KdV-type equation with multiple soliton solutions: Painlevé analysis and Hirota Bilinear technique. Phys Scr. 2022; 97:125214.

[6] Guan X, Liu W, Zhou Q et al. Darboux transformation and analytic solutions for a generalized super-NLS-mKdV equation. Nonlinear Dyn. 2019; 98:1491–1500.

[7] Li HM, Tian B, Xie XY. Soliton and rogue-wave solutions for a $(2+1)$-dimensional fourth-order nonlinear Schrödinger equation in a Heisenberg ferromagnetic spin chain. Nonlinear Dyn. 2016; 86:369–380.

[8] Lan ZZ. Rogue wave solutions for a higher-order nonlinear Schrödinger equation in an optical fiber. Appl Math Letters.2020; 107:106382.

[9] Wazwaz AM. Multiple soliton solutions for a (2+1)-dimensional integrable KdV6 equation. Commun. Nonlinear Sci. Numer. Simul. 2010; 15:1466–1472.

[10] Baldwin D, Hereman W. Symbolic software for the Painlevé test of nonlinear differential ordinary and partial equations. Journal of Nonlinear Mathematical Physics. 2006; 13(1):90-110.

[11] Hereman W, Nuseir A. Symbolic methods to construct exact solutions of nonlinear partial differential equations. Math Comput Simul, 1997; 43:13–27.

[12] Huang ZR, Tian B, Zhen HL et al. Bäcklund transformations and soliton solutions for a (3+1)-dimensional B-type Kadomtsev–Petviashvili equation in fluid dynamics. Nonlinear Dyn. 2015; 80:1–7.

[13] Yan XW, Tian SF, Dong MJ et al. Bäcklund transformation, rogue wave solutions and interaction phenomena for a (3+1)-dimensional B-type Kadomtsev–Petviashvili–Boussinesq equation. Nonlinear Dyn. 2018; 92:709–720.

[14] Kumar S, Kumar D, Wazwaz AM. Lie symmetries, optimal system, group-invariant solutions and dynamical behaviors of solitary wave solutions for a (3+1)-dimensional KdV-type equation, Eur Phys J Plus. 2021; 136:531.

[15] Kumar S, Niwas M. New optical soliton solutions and a variety of dynamical wave profiles to the perturbed Chen–Lee–Liu equation in optical fibers. Opt Quant Electron. 2023; 55:418.

[16] Dhiman SK, Kumar S. An optimal system, invariant solutions, conservation laws, and complete classification of Lie group symmetries for a generalized (2+1)-dimensional Davey–Stewartson system of equations for the wave propagation in water of finite depth. Eur Phys J Plus. 2023; 138:195.

[17] Verma P, Pandit S, Kumar M et al. Time-fractional (2+1)-dimensional navier-stokes equations: similarity reduction and exact solutions for one-parameter lie group of rotations Phys. Scr. 2023; 98:075233.

[18] Verma P, Kumar V, Kumar M, Poonam. Perturbed Fokas–Lenells equation: Lie symmetry analysis, complexitons and baseband modulation instability, International Journal of Modern Physics B, 2023; 37(2):2350015.

[19] Kumar V, Jiwari R, Djurayevich AR, Khudoyberganov, MU. Hyperbolic (2+1)-dimensional Schrödinger equation: Similarity analysis, Optimal system and complexitons for the one-parameter group of rotations, Communications in Nonlinear Science and Numerical Simulation, 2022; 115:106784.

[20] Asaad MG, Ma WX. Pfaffian solutions to a $(3+1)$-dimensional generalized B-type Kadomtsev–Petviashvili equation and its modified counterpart. Appl. Math. Comput. 2012; 218:5524–5542.

[21] Huang QM, Gao YT. Wronskian, Pfaffian and periodic wave solutions for a (2+1)-dimensional extended shallow water wave equation. Nonlinear Dyn. 2017; 89:2855–2866.

[22] Kravchenko VV. Inverse Scattering Transform Method in Direct and Inverse Sturm-Liouville Problems. Frontiers in Mathematics. Birkhäuser Cham; 2020.

[23] Zhou X. Inverse scattering transform for the time dependent Schrödinger equation with applications to the KPI equation. Commun Math Phys. 1990; 128:551–564.

[24] Nikolkina I, Didenkulova I. Rogue waves in 2006–2010. Nat Hazards Earth Syst. Sci. 2011; 11:2913–2924.

[25] Aly R. Seadawy, Syed T.R. Rizvi, Sarfaraz Ahmed, Azhar Bashir. Lump solutions, Kuznetsov–Ma breathers, rogue waves and interaction solutions for magneto electroelastic circular rod. Chaos Soli Fract 2022; 163:112563.

[26] Residori S, Onorato M, Bortolozzo U, Arecchi FT. Rogue waves: a unique approach to multidisciplinary physics, Contemporary Physics. 2017; 58(1):53-69.

[27] Lingfei Li, Yingying Xie. Rogue wave solutions of the generalized (3+1)-dimensional Kadomtsev–Petviashvili equation. Chaos Soli Fract. 2021; 147:110935.

[28] Yan Sun, Bo Tian, Lei Liu, Xiao-Yu Wu. Rogue waves for a generalized nonlinear Schrödinger equation with distributed coefficients in a monomode optical fiber. Chaos Solit Fract. 2018; 107:266-274.

[29] Mao JJ, Tian SF, Zhang TT. (2019), Rogue waves, homoclinic breather waves and soliton waves for a (3+1)-dimensional non-integrable KdV-type equation, Int. J. Num. Meth. Heat Fluid Flow, 2019; 29(2):763-772.

[30] Kumar S, Mohan B. A direct symbolic computation of center-controlled rogue waves to a new Painlevé-integrable (3+1)-D generalized nonlinear evolution equation in plasmas, Nonlinear Dyn, 2023; https://doi.org/10.1007/s11071-023-08683-5

[31] Zhaqilao: A symbolic computation approach to constructing rogue waves with a controllable center in the nonlinear systems, Comput Math Appli. 2018; 75(9):3331-3342.

[32] Cao Y, Tian H, Ghanbari B. On constructing of multiple rogue wave solutions to the (3+1)-dimensional Korteweg–de Vries Benjamin-Bona-Mahony equation. Phys. Scr. 2021; 96:035226.

[33] Yang X, Zhang Z, Wazwaz AM, Wang Z. A direct method for generating rogue wave solutions to the (3+1)-dimensional Korteweg-de Vries Benjamin-Bona-Mahony equation. Physics Letters A. 2022; 449:128355.

[34] Tariq KUH, Seadawy AR. Soliton solutions of (3+1)-dimensional Korteweg-de Vries Benjamin-Bona-Mahony, Kadomtsev-Petviashvili Benjamin-Bona-Mahony and modified Korteweg de Vries-Zakharov-Kuznetsov equations and their applications in water waves. Journal of King Saud University-Science. 2019; 31(1):8-13.

[35] Ma WX. Lump solutions to the Kadomtsev-Petviashvili equation, Physics Letters A. 2015; 379(36):1975-1978.

[36] Liu JG, Eslami M, Rezazadeh H, et al. Rational solutions and lump solutions to a non-isospectral and generalized variable-coefficient Kadomtsev–Petviashvili equation. Nonlinear Dyn. 2019; 95:1027–1033.

[37] Yin Y, Tian B, Chai HP, et al. Lumps and rouge waves for a (3+1)-dimensional variable-coefficient Kadomtsev-Petviashvili equation in fluid mechanics. Pramana J Phys. 2018; 91:43.

[38] Batwa S, Ma WX. Lump solutions to a generalized Hietarinta-type equation via symbolic computation. Front. Math. China 2020; 15:435–450.

[39] Xu H, Ma Z, Fei J, et al. Novel characteristics of lump and lump–soliton interaction solutions to the generalized variable-coefficient Kadomtsev–Petviashvili equation. Nonlinear Dyn. 2019; 98:551–560.

[40] Mao JJ, Tian SF, Yan XJ, Zhang TT. Lump solutions and interaction phenomena of the (3+1)-dimensional nonlinear evolution equations, Int. J. Num. Meth. Heat Fluid Flow, 2019; 29(9):3417-3436.

[41] Mao JJ, Tian SF, Zou L et al. Bilinear formalism, lump solution, lumpoff and instanton/rogue wave solution of a (3+1)-dimensional B-type Kadomtsev–Petviashvili equation. Nonlinear Dyn, 2019; 95:3005–3017.

[42] Manakov SV, Zakharov VE, Bordag LA, Its AR, Matveev VB. Two-dimensional solitons of the Kadomtsev-Petviashvili equation and their interaction. Physics Letters A. 1977; 63(3):205-206.

[43] Satsuma J, Ablowitz MJ. Two-dimensional lumps in nonlinear dispersive systems. J Math Phys. 1979; 20:1496.

[44] Kaup DJ. The lump solutions and the Bäcklund transformation for the three-dimensional three-wave resonant interaction J Math Phys. 1981; 22:1176.

[45] Zhang Y, Dong H, Zhang X, Yang H. Rational solutions and lump solutions to the generalized (3+1)-dimensional Shallow Water-like equation. Comp Math Appl. 2017; 73(2):246-252.

[46] Kumar S, Mohan B, Kumar R. Lump, soliton, and interaction solutions to a generalized two-mode higher-order nonlinear evolution equation in plasma physics. Nonlinear Dyn. 2022; 110:693–704.

[47] Manukure S, Zhou Y, Ma WX. Lump solutions to a (2+1)-dimensional extended KP equation. Comp Math Appl. 2018; 75(7):2414-2419.

[48] Liu JG, Zhu WH. Multiple rogue wave, breather wave and interaction solutions of a generalized (3 + 1)-dimensional variable-coefficient nonlinear wave equation. Nonlinear Dyn. 2021; 103:1841–1850.

[49] Liu W, Zhang Y. Multiple rogue wave solutions for a (3+1)-dimensional Hirota bilinear equation. Appl Math Letters. 2019; 98:184-190.

[50] Zhaqilao. Rogue waves and rational solutions of a (3+1)-dimensional nonlinear evolution equation,Physics Letters A. 2013; 377(42):3021-3026.

[51] Liu JG, Zhu WH. Multiple rogue wave solutions for (2+1)-dimensional Boussinesq equation, Chinese Journal of Physics. 2020; 67:492-500.

[52] Guo J, He J, Li M, Mihalache D. Multiple-order line rogue wave solutions of extended Kadomtsev–Petviashvili equation. Math Comp Simul. 2021; 180:251-257.

[53] Zhang HY, Zhang YF. Analysis on the M-rogue wave solutions of a generalized (3+1)-dimensional KP equation. Appl Math Letters. 2020; 102:106145.

[54] Li L, Xie Y, Mei L. Multiple-order rogue waves for the generalized (2+1)-dimensional Kadomtsev–Petviashvili equation, Applied Mathematics Letters, 2021; 117:107079.

[55] Elboree MK. Higher order rogue waves for the (3 + 1)-dimensional Jimbo–Miwa equation. Inter J of Nonlin Sci Numer Simul. 2022; 23:7-8.

[56] Xu GQ, Li ZB. Symbolic computation of the Painlevé test for nonlinear partial differential equations using Maple,Computer Physics Communications. 2004; 161:65–75.