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Partial Differentiation

$diff(f(x,y),x)$

$$\frac{\partial}{\partial x} f(x,y) \tag{1}$$

$diff(x^2+y^2,x)$

$$2\,x \tag{2}$$

$diff(x^2+y^2,y)$

$$2\,y \tag{3}$$

$diff(x^2\,y+y^2\,x,x)+diff(x^2+y^2,x)$

$$2\,x\,y+y^2+2\,x \tag{4}$$

Higher-order derivative

$diff(f(x,y),x\$3)$

$$\frac{\partial^3}{\partial x^3} f(x,y) \tag{5}$$

$diff(f(x,y),x\$3,y\$2)$

$$\frac{\partial^5}{\partial x^3\,\partial y^2} f(x,y) \tag{6}$$

$diff(x^2\,y+y^2\,x,x\$2,y)$

$$2 \tag{7}$$

Defining function first then find partial derivative

$h := (x,y) \rightarrow x^2+y^2$

$$h := (x,y) \mapsto y^2+x^2 \tag{8}$$

$diff(h(x,y),x)$

$$2\,x \tag{9}$$

Application of transformation

$tr := g(x,y)=x^2+y$

$$tr := g(x,y)=x^2+y \tag{10}$$

$$tr := g(x,y)=x^2+y \tag{11}$$

$eqn := diff(g(x,y),x)+diff(g(x,y),x,y)+diff(g(x,y),x,x,x)$

$$eqn := \frac{\partial}{\partial x} g(x,y) + \frac{\partial^2}{\partial x \partial y} g(x,y) + \frac{\partial^3}{\partial x^3} g(x,y) \quad \textbf{(12)}$$

subs(*tr*, *eqn*)

$$\frac{\partial}{\partial x} (x^2+y) + \frac{\partial^2}{\partial x \partial y} (x^2+y) + \frac{\partial^3}{\partial x^3} (x^2+y) \quad \textbf{(13)}$$

simplify(**(13)**)

$$2\,x \quad \textbf{(14)}$$

Numerical Method Calculations

$fsolve(x^2 - 5\,x + 6)$

$$2.000000000,\ 3.000000000 \quad \textbf{(15)}$$

Solving two function equations

$f1 := \sin(x+y) - \exp(x \cdot y) = 0$

$$f1 := \sin(y+x) - e^{x\,y} = 0 \quad \textbf{(16)}$$

$f2 := x^2 - y = 3$

$$f2 := x^2 - y = 3 \quad \textbf{(17)}$$

$fsolve(\{f1, f2\})$

$$\{x = -2.304131741,\ y = 2.309023079\} \quad \textbf{(18)}$$

Or solving multiple polynomials

$ploy := \{x^3 + 5\,y - 3 = 0,\ y^2 + z - 2 = 0,\ x\,y + x\,z = 5\}$

$$ploy := \{x\,y + x\,z = 5,\ x^3 + 5\,y - 3 = 0,\ y^2 + z - 2 = 0\} \quad \textbf{(19)}$$

fsolve(*poly*)

$$0. \quad \textbf{(20)}$$

Numerical Quadrature

with(*Student*[*Calculus1*]) :

ApproximateInt(exp(−*x*), *x* = 0 ..1, *method* = *trapezoid*)

$$\frac{1}{20} + \frac{e^{-\frac{1}{10}}}{10} + \frac{e^{-\frac{1}{5}}}{10} + \frac{e^{-\frac{3}{10}}}{10} + \frac{e^{-\frac{2}{5}}}{10} + \frac{e^{-\frac{1}{2}}}{10} + \frac{e^{-\frac{3}{5}}}{10} + \frac{e^{-\frac{7}{10}}}{10} + \frac{e^{-\frac{4}{5}}}{10} + \frac{e^{-\frac{9}{10}}}{10} + \frac{e^{-1}}{20} \quad \textbf{(21)}$$

evalf(**(21)**)

$$0.6326472383 \quad \textbf{(22)}$$

ApproximateInt(exp(−*x*), *x* = 0 ..1, *method* = *simpson*)

$$\frac{1}{60} + \frac{e^{-\frac{2}{5}}}{30} + \frac{e^{-\frac{1}{2}}}{30} + \frac{e^{-\frac{3}{5}}}{30} + \frac{e^{-\frac{7}{10}}}{30} + \frac{e^{-\frac{4}{5}}}{30} + \frac{e^{-\frac{9}{10}}}{30} + \frac{e^{-\frac{1}{10}}}{30} + \frac{e^{-\frac{1}{5}}}{30} + \frac{e^{-\frac{3}{10}}}{30} \quad \textbf{(23)}$$

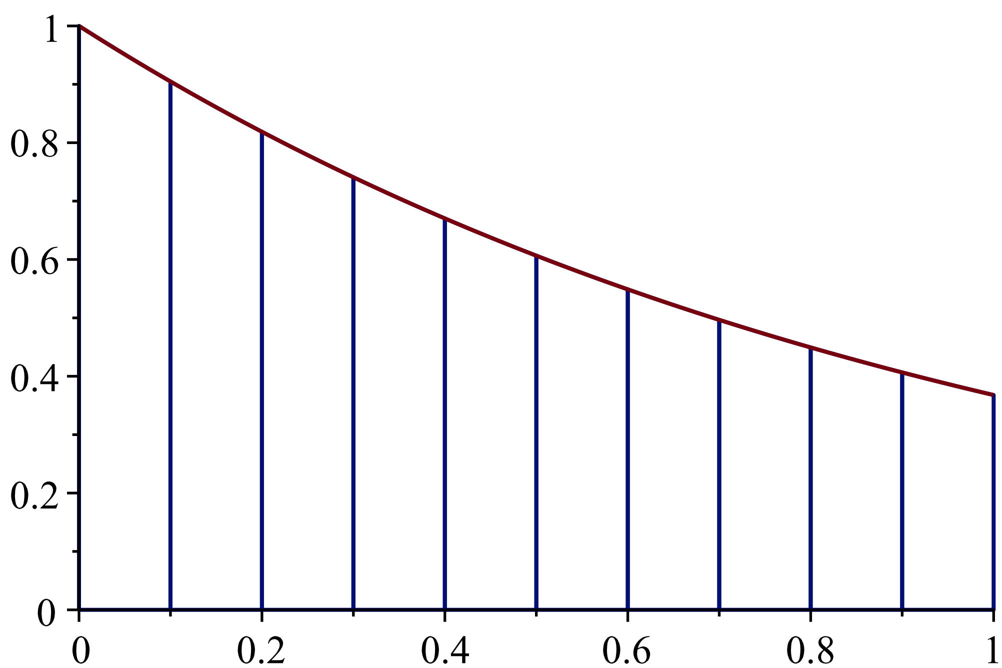
$$+\frac{e^{-1}}{60}+\frac{e^{-\frac{9}{20}}}{15}+\frac{e^{-\frac{11}{20}}}{15}+\frac{e^{-\frac{13}{20}}}{15}+\frac{e^{-\frac{3}{4}}}{15}+\frac{e^{-\frac{1}{20}}}{15}+\frac{e^{-\frac{3}{20}}}{15}+\frac{e^{-\frac{1}{4}}}{15}+\frac{e^{-\frac{7}{20}}}{15}$$
$$+\frac{e^{-\frac{17}{20}}}{15}+\frac{e^{-\frac{19}{20}}}{15}$$

evalf((**23**))

$$0.6321205808 \qquad \textbf{(24)}$$

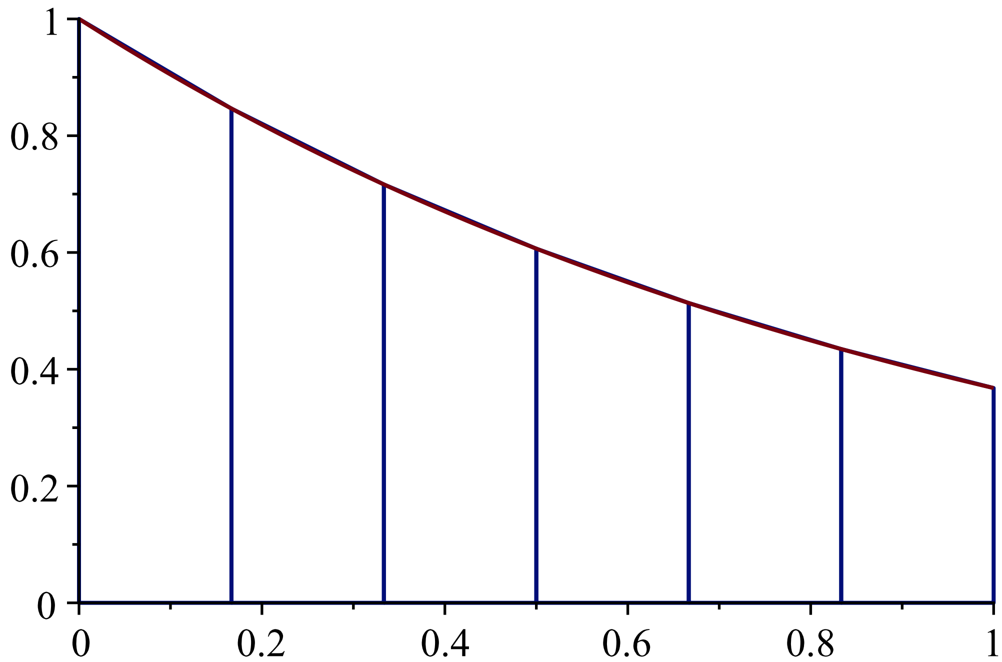
Graphical Approximation

ApproximateInt(exp(−*x*), *x* = 0 ..1, *method* = *simpson*, *output* = *plot*) Having partition by default as 10



An approximation of $\int_0^1 f(x)\, dx$ using Simpson's rule, where $f(x) = e^{-x}$ and the partition is uniform. The approximate value of the integral is 0.6321205808. Number of subintervals used: 10.

ApproximateInt(exp(−*x*), *x* = 0 ..1, *method* = *trapezoid*, *output* = *plot*, *partition* = 6) Having Partition as 6



An approximation of $\int_0^1 f(x)\,dx$ using trapezoid rule, where $f(x) = e^{-x}$ and the partition is uniform. The approximate value of the integral is 0.6335831239. Number of subintervals used: 6.

Maximum and Minimum Value of a function

$\cos(x) \xrightarrow{\text{minimize}} [-1., [x = 3.14159265358977]]$

$\cos(x) \xrightarrow{\text{maximize}} [1., [x = 5.58237824894110\ 10^{-17}]]$

$maximize(\exp(\sin(x)))$

$$e \quad (25)$$

$maximize(\exp(\tan(x)), x = 0..10)$

$$\infty \quad (26)$$

$minimize(x^2 - 3x + y^2 + 3y + 3)$

$$-\frac{3}{2} \quad (27)$$

$minimize(x^2 - 3x + y^2 + 3y + 3, x = -4..-2)$

$$y^2 + 3y + 13 \quad (28)$$

$minimize(x^2 - 3x + y^2 + 3y + 3, x = -4..-2, y = 2..4)$

$$23 \quad (29)$$

Finding location or point

$maximize(\sin(x), x = 1 ..3, location)$

$$1, \left\{\left[\left\{x = \frac{\pi}{2}\right\}, 1\right]\right\} \tag{30}$$

$minimize\left(x^2 - 3\,x + y^2 + 3\,y + 3, x = -4 ..-2, y = 2 ..4, location\right)$

$$23, \{[\{x = -2, y = 2\}, 23]\} \tag{31}$$