

A study of multi-soliton solutions, breather, lumps, and their interactions for Kadomtsev-Petviashvili equation with variable time coefficient using Hirota method

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Abstract: This paper investigates the new KP equation with variable coefficients of time ‘ t ’, broadly used to elucidate shallow water waves that arise in plasma physics, marine engineering, ocean physics, nonlinear sciences, and fluid dynamics. In 2020, Wazwaz [1] proposed two extensive KP equations with time-variable coefficients to obtain several soliton solutions and used Painlevé test to verify their integrability. In light of the research described above, we chose one of the integrated KP equations with time-variable coefficients to obtain multiple solitons, rogue waves, breather waves, lumps, and their interaction solutions relating to the suitable choice of time-dependent coefficients. For this KP equation, the multiple solitons and rogue waves up to fourth-order solutions, breather waves, and lump waves along with their interactions are achieved by employing Hirota’s method. By taking advantage of *Wolfram Mathematica*, the time-dependent variable coefficient’s effect on the newly established solutions can be observed through the three-dimensional wave profiles, corresponding contour plots. Some newly formed mathematical results and evolutionary dynamical behaviors of wave-wave interactions are shown in this work. The obtained results are often more advantageous for the analysis of shallow water waves in marine engineering, fluid dynamics, and dusty plasma, nonlinear sciences, and this approach has opened up a new way to explain the dynamical structures and properties of complex physical models. This study examines to be applicable in its influence on a wide-ranging class of nonlinear KP equations.

Keywords: KP equations; Analytical solutions; Hirota method; Soliton solutions; Rogue waves; Breather waves; Lumps.

1 Introduction

Investigating the integrability of nonlinear evolution equations can help to accomplish the objective of obtaining multiple soliton solutions as well as analytical solutions [1–7]. It is familiar that all integrable nonlinear partial differential equations (NLPDE’s) carry exponentially localized soliton solutions in definite directions. Complete integrability for an NLPDE can be confirmed using the Painlevé test [8]. It becomes very monotonous to examine whether an NLPDE proceeds the Painlevé test, but symbolic system software makes it feasible to conduct such analysis. We seek specific solutions to correctly apprehend the characteristic of varied facts in different aspects of natural sciences. Computer Algebra System (CAS) software such as Mathematica, Maple, Matlab, and others can be helpful to achieve such solutions. In 2006, Hereman et al. [9] introduced a symbolic computation using *Mathematica*, which can execute the standard Painlevé test, which is constructed on the Weiss-Tabor-Carnevale (WTC)-Krushkal method [10]. Many researchers have been

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attracted to nonlinear PDEs with variable coefficients because of their closeness to real-life situations and their ability to generate a wide range of soliton solutions compared to constant coefficients. The dynamical behavior of soliton solutions derived from NLPDE's has been a fascinating area of study due to its potential relevance in demonstrating realistic features in the dimensions of engineering sciences, nonlinear dynamics, and complex physical systems.

To investigate the steady-state of two-dimensional KdV soliton, Kadomtsev–Petviashvili (KP) [11] proposed the equation as in normalized form as

$$(u_t + uu_x + u_{xxx})_x + 3u_{yy} = 0, \quad (1)$$

where u is the wave-amplitude function of (x, y, t) and represents the nonlinear restoring forces and frequency dispersion of water-waves which are weakly in nature; u_{var} denotes partial derivatives with respect to independent variable $var : x, y, t$. This KP equation (1) achieved substantial interest in different dimensions of research due to its integrability and dispersal property. As the equation gives rise to multiple soliton solutions, it finds its comprehensive relevancy in literature, lattice dynamics, optical physics, nonlinear dispersion water-wave modeling, stratified internal waves, and many sciences fields.

Wazwaz [1] studied and formulated two new extended integrable KP equations with variable time coefficients as

$$(u_t + uu_x + u_{xxx})_x + 3u_{yy} + T(x, y, t) = 0, \quad (2)$$

where $T(x, y, t)$ is the additional added term in Eq. (1), which is taken as $g(t)u_{xy}$ or $h(t)u_{xx}$ to formulate the said two equations. Here, u denotes the same wave-amplitude function as in equation (1) and $g(t)$ and $h(t)$ represents the functions of time 't', that are brought about by physical and geometrical inhomogeneities in the form of material density, altering radius and others. We chose the equation (2) with $T(x, y, t) = g(t)u_{xy}$ as

$$(u_t + uu_x + u_{xxx})_x + 3u_{yy} + g(t)u_{xy} = 0, \quad (3)$$

in light of our research described as above to obtain multiple soliton solutions, breather, lumps, and their interactions relating to the suitable choice of time-dependent coefficients. The weak non-linearity and weak dispersion behavior of KP equations with variable-coefficients have caught the attention of several scientific studies, where a range of methodical approaches was implemented to acquire bright multiple solitons [12–17], breather [18–22], lump solutions [23–26] and their interactions [27–29]. Many techniques have been developed to ascertain the exact solutions for the NLPDE's, such as Hirota bilinear method [30–32], Darboux transformation [33–35], simplified Hirota method [9, 36], Bäcklund transformation [37, 38], Lie symmetry analysis [39–42], Pfaffian technique [43, 44], Inverse scattering method [45, 46] and several other techniques.

Hirota bilinear method is one of the most productive, powerful and efficient tools for generating single soliton, multiple soliton solutions, breather, lump solutions of integrable NLPDE's. To obtain the above-stated solutions, one needs to form the Hirota bilinear form from the given NLPDE initially. The answers for lump depict a rational function solution localized in all dimensions compared to soliton solutions. Several studies carry to obtain lump solutions for many integrable equations. Taking advantage of symbolic system software *Mathematica*, the Hirota bilinear method can provide us with profoundly feasible lump solutions to NLPDE's.

The primary goal of this work is to obtain multiple soliton solutions, rogue waves, breathers, lump solutions, and their interactions for the integrable KP equation with time-dependent coefficients, as well as a secondary goal of exhibiting the evolutionary dynamics of obtained soliton solutions by taking the best suitable values of arbitrary functions and constants through three-dimensional graphics and corresponding contour plots.

The rest of the article is structured as follows: In section 2, we briefly discuss the Painlevé analysis for Eq. (3). In section 3, we developed a bilinear form using Hirota's method. Section 4 illustrates soliton solutions and rogue waves up to fourth order, breathers, and their interaction solutions, along with their graphical depictions. In section 5, we derive first and second-order lump solutions followed by comparison and discussion in section 6. Finally, we present our findings and conclusions in the last section.

2 Painlevé analysis

Complete integrability of the equation (3) for any function $g(t)$ of time t can be validated using Painlevé test and WTC-Krushal method as discussed in [10, 15, 47]. It provides a characteristics equation for the resonances values of $k = -1, 4, 5$ and 6 , where $k = -1$ conforms to the arbitrary singular manifold $\phi(x, t)$ in assumed solution as a Laurent expansion in [1], depicted as

$$u(x, y, t) = \sum_{i=0}^{\infty} u_i(x, y, t) \phi^{i-\delta}, \quad (4)$$

where the resultant outcomes of $\delta = 1$ and $u_0 = \phi(x, y)$ can be obtained by substituting the equation (4) into (3) and equating the most dominant terms.

3 Hirota method: bilinear form

In order to examine multiple soliton solutions, the Hirota method is applied. It is standard procedure to find out bilinear form of equation (3) as suggested by Hirota [30]. By substituting

$$u = e^{\alpha_i}, \quad (5)$$

into the linear terms of Eq. (3), where α_i ($i = 1, 2, 3, \dots$) is the phase variable

$$\alpha_i = k_i x + r_i y - w_i(t), \quad (6)$$

with k_i, r_i as constants and w_i are functions of t . The ascertained dispersion value of $w_i(t)$ as

$$w_i(t) = \int \frac{k_i^4 + g(t)k_i r_i + 3r_i^2}{k_i} dt = \int \left(k_i^3 + g(t)r_i + \frac{3r_i^2}{k_i} \right) dt, \quad (7)$$

is found to be dependent on the time dependent coefficient function $g(t)$ of equation (3). Assuming the dependent variable transformation

$$u = R(\ln f)_{xx}, \quad (8)$$

where R is a non-zero constant and $f = f(x, y, t)$ is an auxiliary function, which will be determined later. To determine the value of R in (8), we choose

$$f = 1 + e^{\alpha_i} = 1 + e^{k_i x + r_i y - w_i(t)}. \quad (9)$$

On substituting equations (8) and (9) into (3), we get the solution of $R = 0$ and 12 . Since R can not be zero, we consider $R = 12$ as an appropriate choice which is independent of the function $g(t)$. Thus, the transformation turns out to be

$$u(x, y, t) = 12(\ln f)_{xx} = 12 \frac{f f_{xx} - f_x^2}{f^2}. \quad (10)$$

On utilizing the above transformation (10) into equation (3), we developed the appropriate elucidated interpreted form of equation (3) as

$$3f_{xx}^2 - 3f_y^2 + 3ff_{yy} - f_x f_t - g(t)f_x f_y + ff_{xt} + g(t)ff_{xy} - 4f_x f_{xx} + ff_{xxx} = 0, \quad (11)$$

which is a quadratic equation and gives the desired Hirota's bilinear form of equation (3):

$$(D_x^4 + g(t)D_x D_y + D_x D_t + 3D_y^2)f \cdot f = 0, \quad (12)$$

where D represents the bilinear operator designed and defined by Hirota [30].

4 Soliton solutions and breather along with their interactions

4.1 1-Soliton solution

Considering the function f in Eq. (12) as

$$f(x, y, t) = f_1 = 1 + e^{\alpha_1} = 1 + e^{k_1 x + r_1 y - w_1(t)}. \quad (13)$$

The following can be clearly stated from equation (13)

$$f_x = k_1 e^{k_1 x + r_1 y - w_1(t)}, \quad (14)$$

$$f_{xx} = k_1^2 e^{k_1 x + r_1 y - w_1(t)}. \quad (15)$$

Thus, by putting up the values from (13), (14) and (15) into equation (10), we obtain a single soliton solution which is significantly marked by noteworthy solitons as stated below:

$$u(x, y, t) = 12 \left(\frac{e^{\alpha_1} k_1^2}{1 + e^{\alpha_1}} - \frac{e^{2\alpha_1} k_1^2}{(1 + e^{\alpha_1})^2} \right) = 12 k_1^2 \frac{e^{k_1 x + r_1 y - w_1(t)}}{(e^{w_1(t)} + e^{k_1 x + r_1 y})^2}. \quad (16)$$

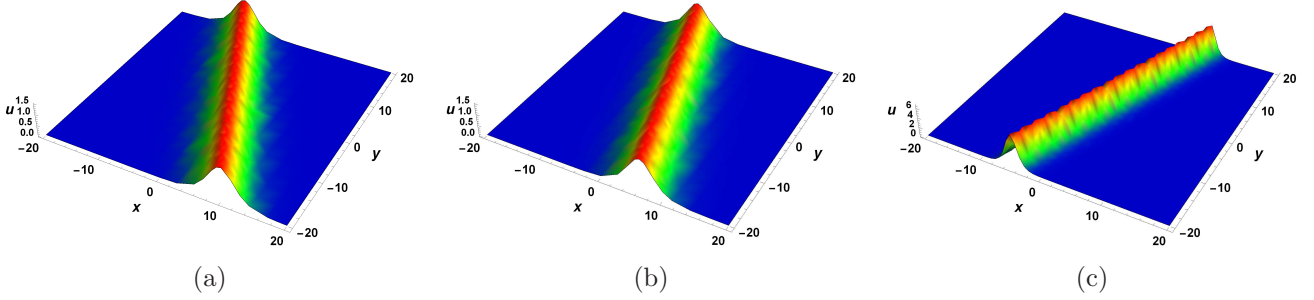


Figure 1: 1-soliton solution via (10) for the function (13) with values: **(a)** $g(t) = t, t = 1, k_1 = 0.70, r_1 = 0.30$, **(b)** $g(t) = \sin(t), t = \pi/3, k_1 = 0.80, r_1 = 0.20$, and **(c)** $g(t) = e^t, t = 1, k_1 = 1.50, r_1 = 0.50$.

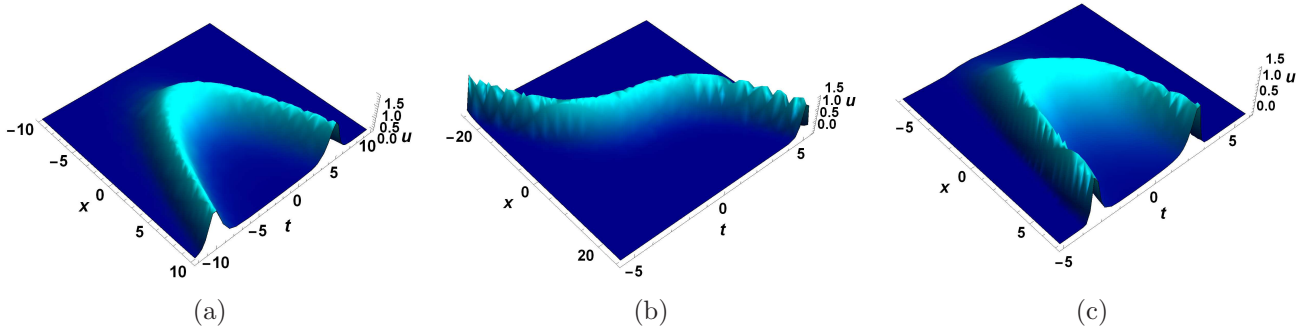


Figure 2: Evolution of the first-order rogue waves via (10) for the function (13) with values $y = 1, k_1 = 0.70, r_1 = 0.30$: **(a)** $g(t) = t$, **(b)** $g(t) = t^2$, and **(c)** $g(t) = t^3$.

4.2 2-Soliton solution

For accomplishing the 2-soliton solution, we select function f as

$$f(x, y, t) = f_2 = 1 + e^{\alpha_1} + e^{\alpha_2} + a_{12}e^{\alpha_1 + \alpha_2}, \quad (17)$$

where a_{12} is the dispersion coefficient which can be determined by substituting f and its derivatives from (17) into the bilinear form (12). Symbolic computations resolve the values of a_{12} , as showcased below:

$$a_{12} = \frac{k_2^2(k_1^2(k_1 - k_2)^2 - r_1^2) + 2k_1k_2r_1r_2 - k_1^2k_2^2}{k_2^2(k_1^2(k_1 + k_2)^2 - r_1^2) + 2k_1k_2r_1r_2 - k_1^2k_2^2}, \quad (18)$$

which can be extrapolated for the auxiliary function

$$f(x, y, t) = 1 + e^{\alpha_i} + e^{\alpha_j} + a_{ij}e^{\alpha_i + \alpha_j}, \quad (19)$$

as

$$a_{ij} = \frac{k_j^2(k_i^2(k_i - k_j)^2 - r_i^2) + 2k_i k_j r_i r_j - k_i^2 k_j^2}{k_j^2(k_i^2(k_i + k_j)^2 - r_i^2) + 2k_i k_j r_i r_j - k_i^2 k_j^2}, \quad 1 \leq i < j \leq N, \quad (20)$$

where N is an integer. Thus, from (17) we get

$$f_x = k_1 e^{\alpha_1} + k_2 e^{\alpha_2} + a_{12}(k_1 + k_2)e^{\alpha_1 + \alpha_2}, \quad (21)$$

$$f_{xx} = k_1^2 e^{\alpha_1} + k_2^2 e^{\alpha_2} + a_{12}(k_1 + k_2)^2 e^{\alpha_1 + \alpha_2}. \quad (22)$$

The substitution of (17), (21) and (22) into (10), yields a second order soliton solution of equation (3).

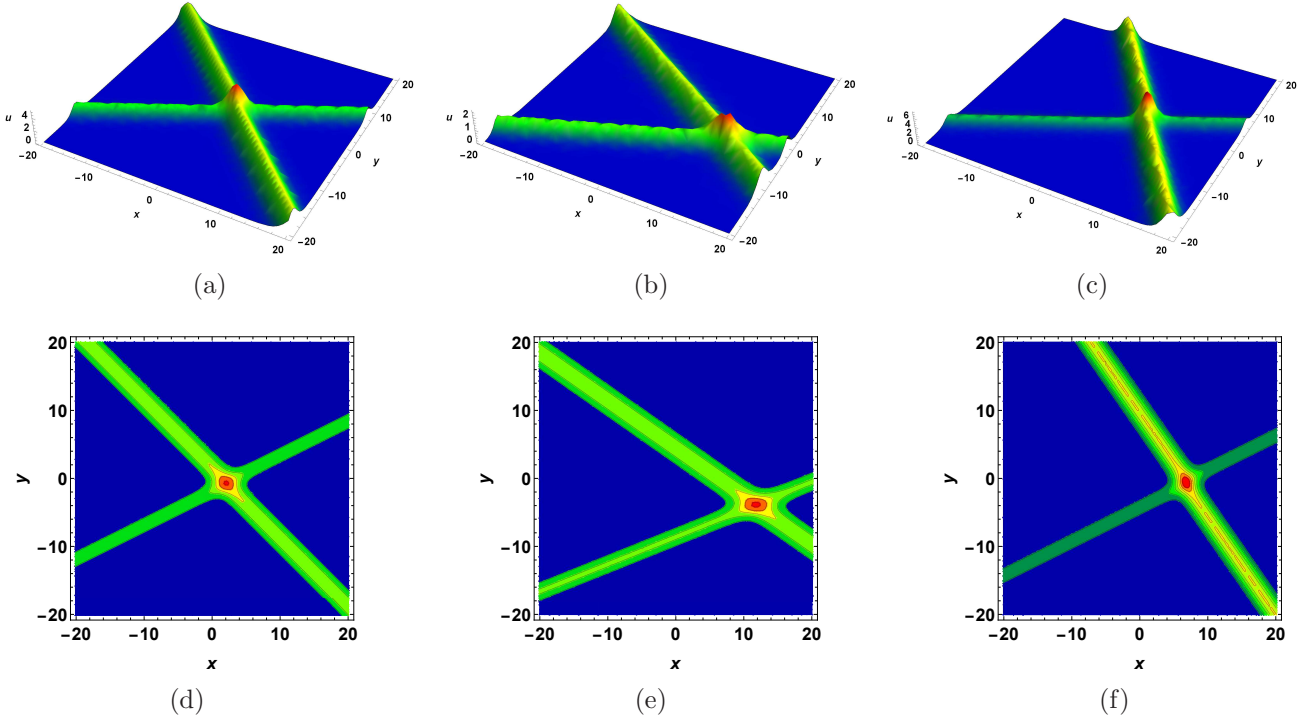


Figure 3: 2-soliton interaction solution via (10) for the function (17) with values: **(a)** $g(t) = t, t = 0.30, k_1 = 1, k_2 = -0.75, r_1 = 1, r_2 = 1.50$, **(b)** $g(t) = \sin(t), t = \pi/3, k_1 = 0.70, k_2 = -0.60, r_1 = 1, r_2 = 1.50$, and **(c)** $g(t) = e^t, t = 1.20, k_1 = 1.3, k_2 = -0.75, r_1 = 0.9, r_2 = 1.50$. **(d-f)** portray contour plots of **(a-c)** w.r.t. xy -plane.

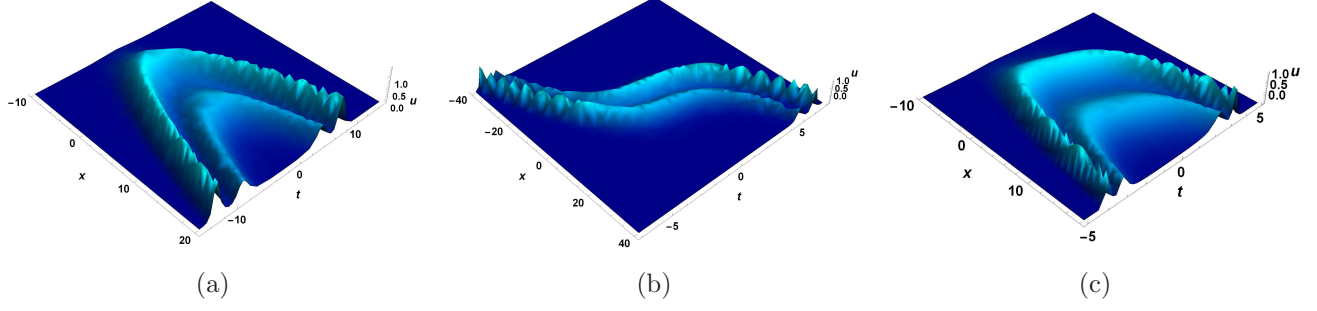


Figure 4: Evolution of the second-order rogue waves via (10) for the function (17) with values $y = 1, k_1 = 0.70, k_2 = 0.5, r_1 = 0.30, r_2 = 0.30$: (a) $g(t) = t$, (b) $g(t) = t^2$, and (c) $g(t) = t^3$.

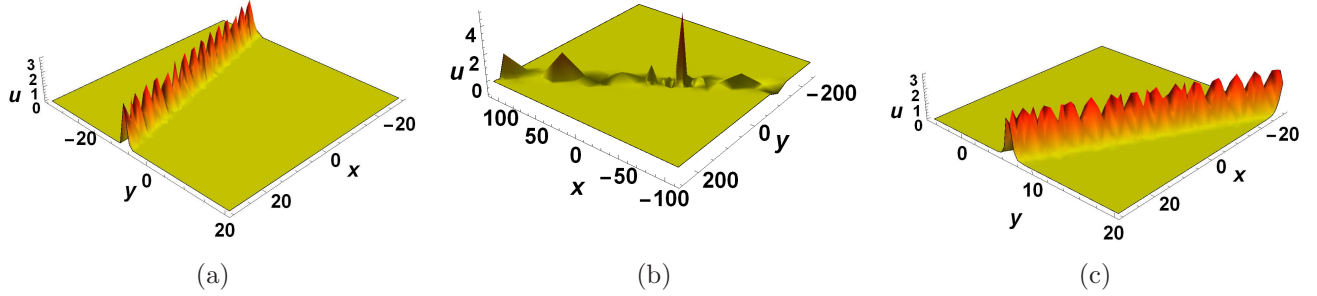


Figure 5: Single-breather solutions via (10) for the function (17) with values ($i = \sqrt{-1}$): (a) $g(t) = t, t = 1, k_1 = k_2 = -1, r_1 = r_2 = 4.5$, (b) $g(t) = t^2, t = 1, k_1 = k_2 = -0.3, r_1 = 0.2 + i, r_2 = 0.2 - i$, and (c) $g(t) = \sin(t), k_1 = k_2 = 1, r_1 = r_2 = 4.5$.

4.3 3-Soliton solution

The 3-soliton solution can be constructed using the assumed function f as

$$f(x, y, t) = f_3 = 1 + e^{\alpha_1} + e^{\alpha_2} + e^{\alpha_3} + a_{12}e^{\alpha_1+\alpha_2} + a_{13}e^{\alpha_1+\alpha_3} + a_{23}e^{\alpha_2+\alpha_3} + b_{123}e^{\alpha_1+\alpha_2+\alpha_3}, \quad (23)$$

where a_{ij} with $1 \leq i < j \leq 3$ fulfills the relation (20) and the dispersion coefficient b_{123} can be validated by making use of symbolic computation to favor the following constraint:

$$b_{123} = a_{12}a_{13}a_{23}, \quad (24)$$

which can also be envisioned for the auxiliary function

$$f = 1 + e^{\alpha_l} + e^{\alpha_m} + e^{\alpha_n} + a_{lm}e^{\alpha_l+\alpha_m} + a_{ln}e^{\alpha_l+\alpha_n} + a_{mn}e^{\alpha_m+\alpha_n} + b_{lmn}e^{\alpha_l+\alpha_m+\alpha_n}, \quad (25)$$

where the third-order dispersion coefficient b_{lmn} justifies

$$b_{lmn} = a_{lm}a_{ln}a_{mn}; \quad 1 \leq l < m < n \leq N, \quad (26)$$

where $a_{ij}; 1 \leq i < j \leq N$ is deduced from equation (20). Therefore, from (23), we have

$$f_x = \sum_{p=1}^3 k_p e^{\alpha_p} + \sum_{1 \leq p < q \leq 3} (k_p + k_q) a_{pq} e^{\alpha_p + \alpha_q} + \left(\sum_{p=1}^3 k_p \right) b_{123} e^{\alpha_1 + \alpha_2 + \alpha_3}, \quad (27)$$

$$f_{xx} = \sum_{p=1}^3 k_p^2 e^{\alpha_p} + \sum_{1 \leq p < q \leq 3} (k_p + k_q)^2 a_{pq} e^{\alpha_p + \alpha_q} + \left(\sum_{p=1}^3 k_p \right)^2 b_{123} e^{\alpha_1 + \alpha_2 + \alpha_3}. \quad (28)$$

By substituting the values of (23), (27) and (28) into (10), we establish the 3-soliton solution.

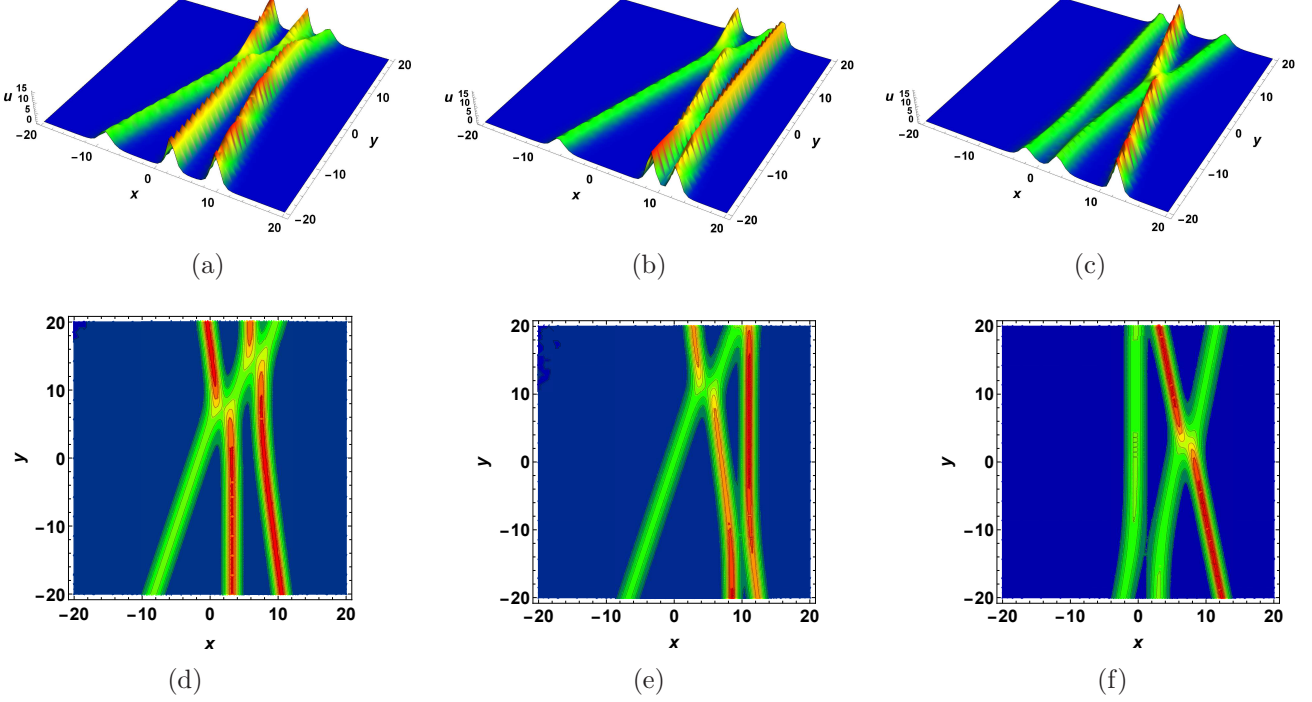


Figure 6: 3-soliton interaction solution via (10) for the function (23) with values: **(a)** $g(t) = t, t = 1, k_1 = 1.50, k_2 = -2.25, k_3 = -2.20, r_1 = -0.50, r_2 = -0.30, r_3 = 0$, **(b)** $g(t) = \sin(t), t = \pi/2, k_1 = 1.50, k_2 = -2.25, k_3 = -2.50, r_1 = -0.50, r_2 = -0.30, r_3 = 0.01$, and **(c)** $g(t) = e^t, t = 1, k_1 = 2.50, k_2 = -1.50, k_3 = 1.70, r_1 = 0.50, r_2 = 0.30, r_3 = 0$. **(d-f)** portray contour plots of **(a-c)** w.r.t xy -plane.

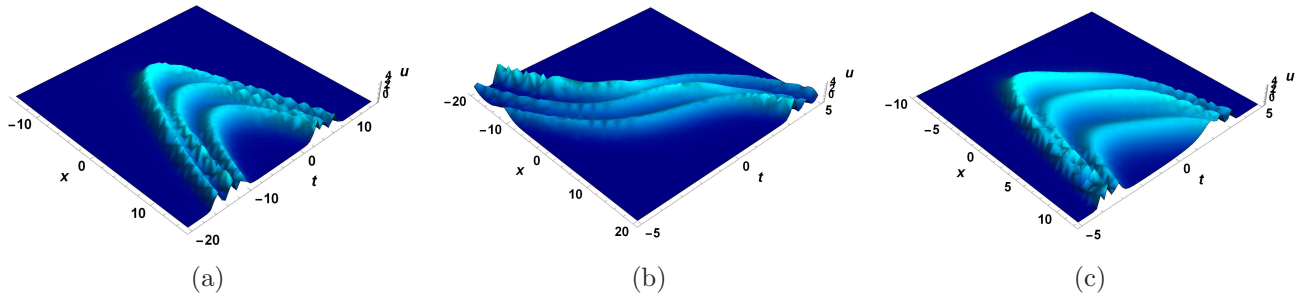


Figure 7: Evolution of the third-order rogue waves via (10) for the function (23) with values $y = 1, k_1 = 1.40, k_2 = 1, k_3 = 1, r_1 = 0.70, r_2 = 0.40, r_3 = 0.3$: **(a)** $g(t) = t$, **(b)** $g(t) = t^2$, and **(c)** $g(t) = t^3$.

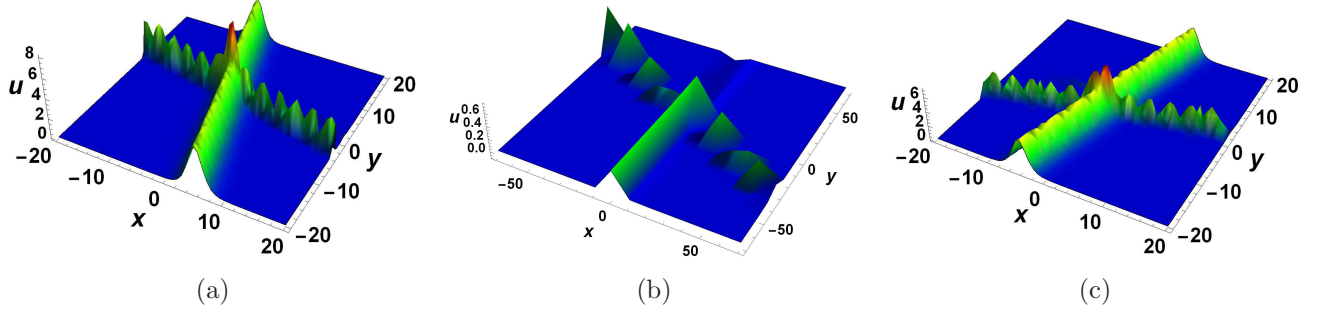


Figure 8: 1-soliton and 1-breather interaction solution via (10) for the function (23) with values ($i = \sqrt{-1}$): (a) $g(t) = t, t = 0.10, k_1 = 1.20, k_2 = 1.10, k_3 = 0.42, r_1 = 0.30, r_2 = r_3 = 5$, (b) $g(t) = t^2, t = 0.30, k_1 = 0.50, k_2 = 1 + 0.50i, k_3 = 1 - 0.50i, r_1 = 0.90, r_2 = r_3 = 0$, (c) $g(t) = \sin(t), t = 0.10, k_1 = -1.20, k_2 = -1.10, k_3 = 0.42, r_1 = 0.30, r_2 = r_3 = 5$.

4.4 4-Soliton solution

Taking into account the 4-soliton solution function f as

$$f(x, y, t) = f_4 = 1 + \sum_{p=1}^4 e^{\alpha_p} + \sum_{1 \leq p < q \leq 4} a_{pq} e^{\alpha_p + \alpha_q} + \sum_{1 \leq l < m < n \leq 4} b_{lmn} e^{\alpha_l + \alpha_m + \alpha_n} + c_{1234} e^{\sum_{i=1}^4 \alpha_i}, \quad (29)$$

where $a_{ij}; 1 \leq i < j \leq 4$ and $b_{lmn}; 1 \leq l < m < n \leq 4$ satisfy the constraints (20) and (26), respectively. With the help of symbolic computation, we can retrieve the 4th-order dispersion coefficient c_{1234} which must satisfy the following expression:

$$c_{1234} = a_{12}a_{13}a_{14}a_{23}a_{24}a_{34}. \quad (30)$$

From (29), we can mark out

$$f_x = \sum_{p=1}^4 k_p e^{\alpha_p} + \sum_{1 \leq p < q \leq 4} (k_p + k_q) a_{pq} e^{\alpha_p + \alpha_q} + \sum_{1 \leq l < m < n \leq 4} (k_l + k_m + k_n) b_{lmn} e^{\alpha_l + \alpha_m + \alpha_n} + \left(\sum_{p=1}^4 k_p \right) c_{1234} e^{\sum_{p=1}^4 \alpha_p}, \quad (31)$$

$$f_{xx} = \sum_{p=1}^4 k_p^2 e^{\alpha_p} + \sum_{1 \leq p < q \leq 4} (k_p + k_q)^2 a_{pq} e^{\alpha_p + \alpha_q} + \sum_{1 \leq l < m < n \leq 4} (k_l + k_m + k_n)^2 b_{lmn} e^{\alpha_l + \alpha_m + \alpha_n} + \left(\sum_{p=1}^4 k_p \right)^2 c_{1234} e^{\sum_{p=1}^4 \alpha_p}. \quad (32)$$

On substituting (29), (31) and (32) into equation (10), leads to resultant fourth-order soliton solution.

5 Lumps: 1-lump and 2-lump solutions

In order to accomplish the 1-lump solution of Eq. (3), it is necessary to find a satisfactory rational auxiliary function f .

Considering the function f for 1st-order lump solution as

$$f = f_5 = \alpha_1 \alpha_2 + d_{12}, \quad (33)$$

where the values of α_i are specified in (20) and d_{12} is the dispersion coefficient which is required to be resolved.

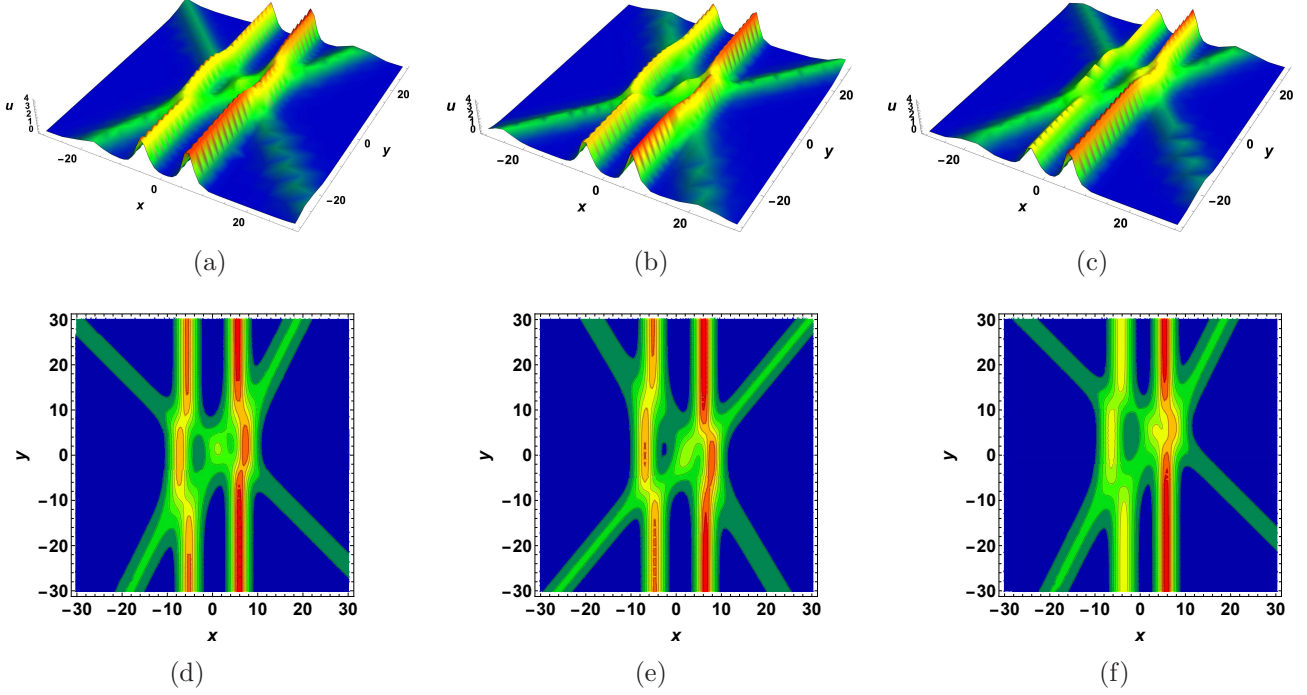


Figure 9: 4-soliton interaction solutions via (10) for the function (29) with values: (a) $g(t) = t^2, t = 0.50, k_1 = -0.60, k_2 = 0.50, k_3 = -1.20, k_4 = 1.10, r_1 = 0.30, r_2 = 0.50, r_3 = 0, r_4 = 0$, (b) $g(t) = \sin(t), t = \pi/4, k_1 = -0.60, k_2 = 0.50, k_3 = -1.20, k_4 = 1.10, r_1 = 0.50, r_2 = 0.30, r_3 = 0, r_4 = 0$, and (c) $g(t) = \log(t), t = 1.20, k_1 = -0.60, k_2 = 0.50, k_3 = -1.20, k_4 = 1, r_1 = 0.30, r_2 = 0.50, r_3 = 0, r_4 = 0$. (d-f) portray contour plots of (a-c) w.r.t xy -plane.

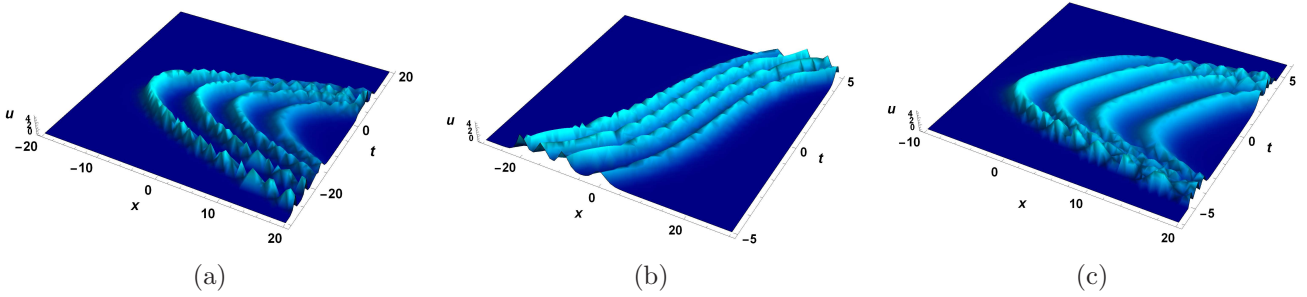


Figure 10: Evolution of the fourth-order rogue waves via (10) for the function (29) with values $y = 1, k_1 = 1, k_2 = 1.20, k_3 = 1.30, r_4 = 1.40, r_1 = 0.20, r_2 = 0.30, r_3 = 0.30, r_4 = 0.30$: (a) $g(t) = t$, (b) $g(t) = t^2$, and (c) $g(t) = t^3$.

On substituting f and its derivatives from (33) into Eq. (12), we determine the value of d_{12} to get:

$$d_{12} = \frac{k_1 k_2 (k_1^2 k_2^2 (12 + x^2 k_2^2) - 2x k_1 k_2^4 (w_1(t) - y r_1) + k_2^4 (w_1(t) - y r_1)^2 + k_1^4 (-w_2(t) + x k_2 + y r_2)^2)}{k_1^4 k_2^2 + 3k_2^2 r_1^2 - 6k_1 k_2 r_1 r_2 + k_1^2 (k_2^4 + 3r_2^2)}, \quad (34)$$

which can also be deduced for the auxiliary function

$$f = \alpha_i \alpha_j + d_{ij}, \quad (35)$$

as

$$d_{ij} = \frac{k_i k_j (k_i^2 k_j^2 (12 + x^2 k_j^2) - 2x k_i k_j^4 (w_i(t) - y r_i) + k_j^4 (w_i(t) - y r_i)^2 + k_i^4 (-w_j(t) + x k_j + y r_j)^2)}{k_i^4 k_j^2 + 3k_j^2 r_i^2 - 6k_i k_j r_i r_j + k_i^2 (k_j^4 + 3r_j^2)}. \quad (36)$$

By carrying out the substitution of f and its derivatives f_x and f_{xx} from (33) into equation (10) gives the end result solution for 1-lump.

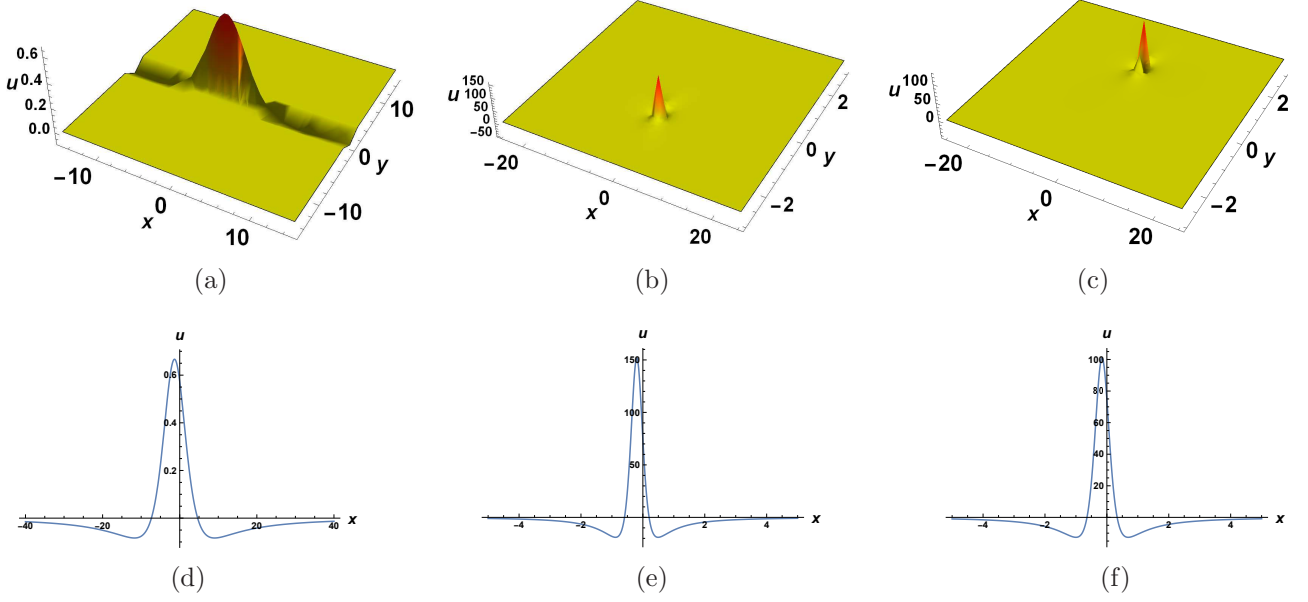


Figure 11: 1-lump solutions via (10) for the function (33) with the values ($i = \sqrt{-1}$): $t = 0.002, k_1 = k_2 = 0.20$ (a) $g(t) = t, r_1 = 4 + 5i, r_2 = 4 - 5i$, (b) $g(t) = \sin(t), r_1 = 0.1 + i, r_2 = 0.1 - i$, and (c) $g(t) = e^t, r_1 = 0.01 + i, r_2 = 0.01 - i$. (d-f) portray the 2d-plots of (a-c), respectively.

To achieve the second order 2-lump solution, we use for the function f as

$$f = f_6 = \alpha_1 \alpha_2 \alpha_3 \alpha_4 + d_{12} \alpha_3 \alpha_4 + d_{13} \alpha_2 \alpha_4 + d_{14} \alpha_2 \alpha_3 + d_{23} \alpha_1 \alpha_4 + d_{24} \alpha_1 \alpha_3 + d_{34} \alpha_1 \alpha_2 + e_{1234}, \quad (37)$$

where $d_{ij}; 1 \leq i < j \leq 4$ assures the relation (36) and dispersion coefficient e_{1234} can be accredited utilizing symbolic computation to compliment the following constraint:

$$e_{1234} = d_{12} d_{34} + d_{13} d_{24} + d_{14} d_{23}. \quad (38)$$

Thus, we obtain a second-order lump solution of equation (3) after substituting f and its derivatives f_x and f_{xx} from (37) into equation (10).

6 Comparison and discussion

In this section, we concisely compare our solutions in the form of multiple solitons, rogue waves, breathers, lumps, and their interactions with the study carried out by the author's work Wazwaz [1], and we deduce the following:

- In Ref. [1] Wazwaz extracted some soliton solutions for the KP equation with time-dependent coefficients. There is no graphical representation of any solution to the same, whereas we have illustrated our solutions with the evolutionary dynamics of soliton solutions through three-dimensional postures.
- Ref. [1] obtained only multiple soliton solutions using the simplified Hirota method, while our research findings showed multiple soliton solutions, rogue waves, breathers, lump solutions, and their interactions using the Hirota bilinear method.

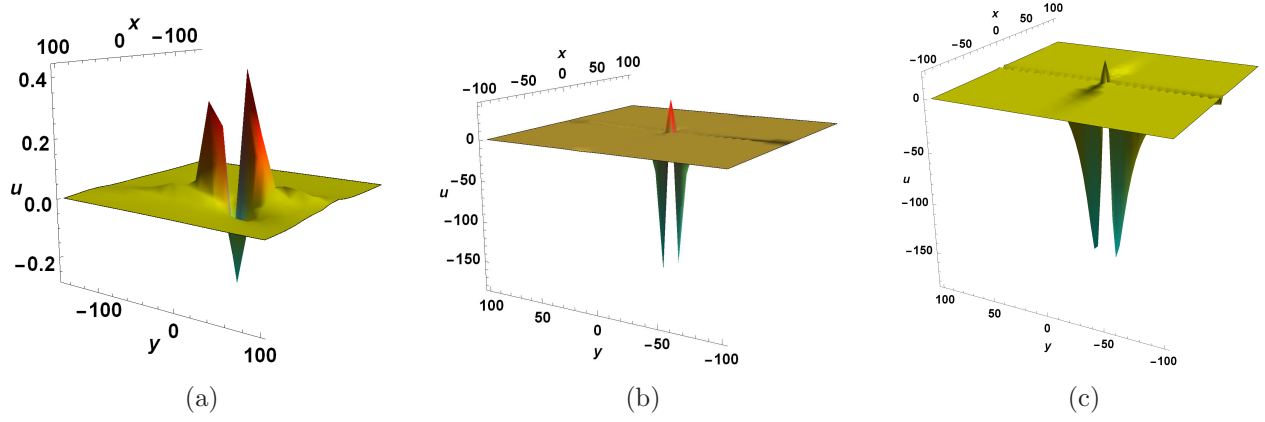


Figure 12: 2-lump solutions via (10) for the function (37) with the values($i = \sqrt{-1}$): **(a)** $g(t) = t, t = 1.30, k_1 = k_2 = 1, k_3 = k_4 = 1.50, r_1 = 0.40 + 0.50i, r_2 = 0.40 - 0.50i, r_3 = 0.25 + 0.50i, r_4 = 0.25 - 0.50i$, **(b)** $g(t) = t^2, t = 0.01, k_1 = 1, k_2 = k_3 = k_4 = 0.50, r_1 = 2.50, r_2 = r_3 = r_4 = 0$, and **(c)** $g(t) = \sin(t), t = 0, k_1 = 3.5, k_2 = k_3 = k_4 = 0.50, r_1 = 2.5, r_2 = r_3 = r_4 = 0$.

- We obtained multiple soliton solutions up to fourth-order whereas ref. [1] came up with third-order multiple soliton solutions.
- We have illustrated our solutions by using both three-dimensional postures and contour plots depending upon the graphical suitability.
- Due to the arbitrariness of the time-dependent coefficient $g(t)$ in equation (3), we have shown graphical representations of the solutions in $x - t - u$ coordination using erratic choices of the same.

7 Conclusions

In this study, we used Hirota's bilinear method to calculate the results of multiple soliton solutions up to fourth-order solutions, rogue waves, breather, lump solutions, and their interactions by making methodical choices of time-dependent coefficients in the integrable Kadomtsev–Petviashvili equation. Subsequently, we demonstrated various dynamical structures by numerically simulating specific values of arbitrary functions and constants. Furthermore, the analytical findings of this study have applicability in terms of their influence on a broad class of nonlinear KP equations and several nonlinear PDEs. In addition, our results show that Hirota's bilinear method is an efficient, robust, and powerful symbolic mathematical tool for solving other higher-dimensional complex physical models in nonlinear sciences, nonlinear dynamics, and engineering physics.

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Conflict of interest

The authors declare that they have no conflict of interest.

Data availability statement

The data that supports the findings of this study are available within the article.

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